c) The Haar wavelet system is defined as

$$\Phi_{0,0} = 1 \qquad 0 \le t \le 1\ ;\ 0 \text{ otherwise}$$

$$\Psi_{0,0} = \begin{cases} 1 & 0 \le t \le ½; \\ -1 & ½ \le t \le 1 \end{cases}$$

and where the two parameters are respectively the scaling and the timing (integer) parameters.
Sketch

$$\Psi_{-1,4},\ \Phi_{0,2},\ \Phi_{2,-1},\ \Psi_{3,3}$$

(8 marks)

3. a) Determine the parameters $\{ K_m\}$ of the lattice filter corresponding to the FIR filter described by the system function

$$H(z) = 1 + 2\,z^{-1} + z^{-2}$$

(5 marks)

b) Determine the zeroes and sketch the zero pattern for the FIR lattice filter with parameters

$$K_1 = ½ \qquad K_2 = -⅓ \qquad K_3 = 1$$

(Hint: You should find that all the zeroes lie on the unit circle)

(8 marks)

Question 3 cont…

4. a) A digital signal is to have a sampling rate conversion using $I = 7$ and $D = 3$. The filter being used, has an impulse response given by $h(n)$ is 35 samples long.
A polyphase filter implementation is to be used.

i) How many polyphase sections are required?
ii) Hence work out the coefficients of $h(n)$ that belong to each polyphase section
iii) Show clearly the order in which the polyphase filters are used to obtain the required output $y(n)$.
iv) Hence show the first eight outputs $y(0)$ to $y(7)$.

(10 marks)

b) A speech signal has a nominal bandwidth of 4 kHz, sampled at 8 kHz. It is required to isolate pitch related frequency components below 200 Hz by a suitable lowpass filter. The filter has a passband between 0 Hz to 160Hz and a transition band from 160Hz to 200 Hz. The allowable passband ripple is $\delta_1 = 10^{-2}$ and the stopband ripple must be below $\delta_2 = 10^{-4}$

i) Calculate the length of the linear phase filter, using Kaiser's formula, to satisfy the requirements.
ii) Calculate the maximum possible decimation factor.
iii) Hence using a suitable two stage decimation that avoids aliasing, calculate the order of the two filters required. State any assumptions made.

(10 marks)

c) A multi resolution dyadic wavelet packet analysis system is to be used to isolate the frequency spectrum of a signal in the range $\pi/19$ to $\pi/24$.
i) Can the required band be obtained exactly? Give reasons.
ii) In any case the exact or approximate band is to be obtained using not more than seven stages of decomposition. Sketch the path through the multiresolution decomposition to obtain the required band or a reasonable approximation.
iii) If the band is not exact work out the percentage difference from the designed to the required band.

(10 marks)

3. (a) **Figure 1** shows a two-channel Quadrature Mirror Filter (QMF) bank with the corresponding pair of analysis and synthesis sections.

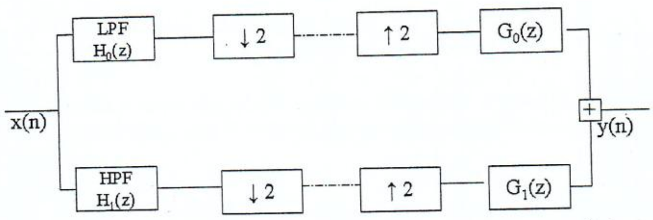

Figure 1

(i) Calculate the resultant output transfer function Y(w) after the interpolator sections in terms of the four filter transfer functions H_0, H_1, G_0, G_1.

(6 marks)

(ii) Hence obtain the relationship between $H_0(z)$ and $H_1(z)$, $G_0(z)$ and $H_0(z)$ and $G_1(z)$ and $H_1(z)$

(5 marks)

(b) (i) In conjunction with the results above, describe the limitations of using linear phase filters to obtain perfect reconstruction in a multiresolution decomposition and reconstruction system.

(4 marks)

(c) For a Haar wavelet system, sketch

(i) the mother smoothing wavelet $\Phi_{0,0}$

(ii) the mother wavelet $\Psi_{0,0}$

(2 marks)

(iii) Write down the relationship for $\Psi_{j,k}$. Hence sketch

$\Phi_{-2,1}$ $\Psi_{2,0}$ $\Psi_{2,4}$

(6 marks)

Question 3 cont....

Question 3 cont….

(d) A signal is sampled at 20,000 samples per second. The signal consists of frequencies in the range of

$$0 \le F < 600 \text{ Hz}$$

And others in the range

$$2000 \le F < 5000 \text{ Hz}$$

(i) Sketch a suitable dyadic multiresolution decomposition system that acts as a lowpass filter and that gives the lowpass signal band at its output. (6 marks)

(ii) Indicate clearly how many stages of decomposition you are using. (1 mark)

4. (a) A filter h(n) has 30 non-zero samples. A signal is sampled at 40,000 samples per second.

(i) Design a suitable polyphase structure assuming that a maximum decimation factor of 6 is possible. (5 marks)

(ii) For your design, letting the transfer function be the sequence h(0) to h(29), give for each polyphase filter in your design the components of h(n) making up the filter. (5 marks)

(iii) Hence for your design, starting with x(0) sample at the input, work out, in terms of x(n) and h(n), the first three outputs, y(0) to y(2). State clearly the initial conditions. (7 marks)

(b) A signal is sampled at 16kHz. It is required to isolate the frequency components below 160Hz. A filter with a passband $0 \le F \le 150$Hz and a transition band between $150 \le F \le 160$Hz is to be used. The passband ripple, δ_1 is 10^{-2} and the minimum stopband ripple δ_2 is 10^{-4}.

Use the Kaiser Formula for calculating the order of a filter, given by

$$M = \frac{-10\log_{10}(\delta_1\delta_2)-13}{14.6\Delta f}+1$$

(i) Calculate the order of filter necessary for the above specifications. (5 marks)

(ii) Calculate the order when two stages of decimation are used given by $D_1 = 25$ and $D_2 = 2$. (8 marks)