CCE5201 - Advanced Signal Processing

January 2011

Answer any FOUR questions

1 (a) Figure 1 shows a time sequence x(n) for $0 \le n \le 7$. The Discrete Fourier Transform, DFT, of x(n) is given by X(k). A second waveform y(n) has a DFT given by $Y(k) = e^{-j6\pi/8}. X(k)$. Using x(n) in Figure 1, sketch the waveform y(n) for $0 \le n \le 7$. (5 marks)

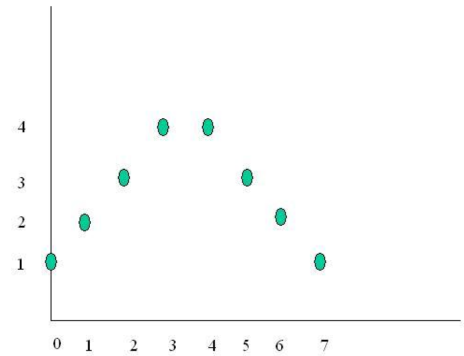

Figure 1

(b) A four point time sequence is given by $x(n) = 1 \quad n=0, 1$
$x(n) = -1 \quad n=2,3$

(i) Work out the four-point DFT of this sequence, X(k) k = 0,1,2,3. Sketch the magnitude response. (7 marks)

(ii) The original sequence x(n) is now augmented by adding four zeroes to obtain an 8-point sequence given by

$$x(n) = 1 \qquad n=0,1 ;$$
$$x(n) = -1 \qquad n=2,3;$$
$$x(n) = 0 \qquad n = 4,5,6,7.$$

Relate the 8-point DFT of the new sequence X'(k) to the DFT, X(k) obtained in b(i) (4 marks)

(c) An FIR filter is to be used on a long sequence. The filter has 50 coefficients. It is decided to use the overlap-save technique with segments of 100 samples. The valid output from one segment must abut that of the adjacent segment in such a way as to produce a continuous valid output. It is decided to use a DFT of 128 points. Numbering the output coefficients from 0 to 127, calculate

(i) the number of overlap samples necessary
(ii) the number of valid output samples
(iii) the index range of the valid output samples.

(4 marks)

(d) A time signal is sampled at 10,000Hz. A set of 128 samples are then passed through a DFT.

(i) What is the resolution in Hz of each DFT sample? (2 marks)

(ii) Numbering the DFT samples from 0 to 127, work out what original frequency is represented by the DFT output sample numbers 25, and 100. (3 marks)

2. (a) A sampling rate converter is required to upsample a signal by a factor of 2.5.A suitable FIR filter having M = 25 coefficients is available. Polyphase structures are to be used to reduce computation.

(i) Determine suitable decimation and interpolation factors. (2 marks)

(ii) Using the filter coefficients from h_0 to h_{24}, work out the number of polyphase filters necessary and the filter coefficients in each polyphase filter. (2 marks)

(iii) Design the system to obtain an output using the minimum number of numerical calculations from the polyphase bank. (3 marks)

(iv) Give the values of the first ten outputs, assuming the initial output is y(0) = x(0). h(0) (6 marks)

(b) A speech signal sequence has been sampled at 20,000 Hz. The sequence is to be lowpass filtered to look at the speech components below 100Hz. A linear phase FIR filter is required having the following specifications

lowpass cutoff frequency 90 Hz
passband ripple limited to 10^{-2}
stopband start frequency 100Hz
stopband attenuation 80dB

Using Kaiser's formula work out the filter order necessary

(i) using one filter (2 marks)

(ii) using a multirate implementation with $D_1 = D_2 = 10$. (2 marks)

(iii) using a multirate implementation with $D_1 = 50$, $D_2 = 2$. (2 marks)

(iv) Would the result in (iii) be the same if $D_1 = 2$, $D_2 = 50$? Give reasons. (2 marks)

(c) Sampling rate conversion uses normally an integer or a rational fraction. Why is this preferred to the use of an arbitrary factor? (4 marks)

3 (a) The Haar transform pair of functions is given by

smoothing function $\Phi(t) = 1 \quad 0 \le t \le 1$
$= 0$ otherwise

and wavelet function $\Psi(t) = 1 \quad 0 \le t \le ½$
$= -1 \quad ½ < t \le 1$
$= 0$ otherwise

The pair are used in a multiresolution decomposition of a discrete wavelet system governed by

$$f_{i,j} = 2^{\frac{i}{2}} f(2^i t - j)$$

Draw the appropriate Haar transform function for the following i,j.

$$\Psi_{0,0}, \quad \Phi_{0,0}, \quad \Psi_{-2,2}, \quad \Phi_{1,1}, \quad \Phi_{2,2}, \quad \Psi_{1,4}.$$

(10 marks)

(b) Describe briefly how a lowpass filter function can be achieved using dyadic discrete wavelet decomposition and reconstruction.

(5 marks)

(c) From a system sampled at 10,000Hz, it is required to isolate the range given by 600Hz to 800Hz. A dyadic wavelet transform multiresolution decomposition is to be used to filter out the signal.

(i) Using not more than 5 decomposition stages, find the nearest approximation to satisfy the requirement. Show clearly the path through the decomposition tree that results in the wavelet packet range you choose. (7 marks)

(ii) Calculate the percentage error in your filter design from the exact required range. (3 marks)

4 (a) What distinguishes the Discrete Cosine Transform from the Discrete Fourier Transform? (5 marks)

(b) Using a decimation in time FFT with 1024 points, and numbering the input array from x(0) to x(1023), calculate the location in the input array of the point x(723) to achieve in place computation. (5 marks)

(c) Sketch an FFT 8-point decimation in time butterfly diagram, showing on the diagram the paths that contribute to the output X(3). (5 marks)

(d) Describe, using an appropriate diagram, the process of homomorphic deconvolution. Mention, giving reasons one application where such processing is appropriate. (5 marks)

(e) Distinguish between a random process, a strict-sense stationary process and a wide-sense stationary process in relation to time sequences.

(5 marks)

5. (a) In sampling Bandpass signals at minimum sampling rate it is necessary to have guard bands to prevent aliasing. Explain why this is necessary.

(5 marks)

(b) An RF signal has a carrier frequency, F_c, of 10.73MHz and a signal bandwidth of 26 kHz. To avoid aliasing, a further guardband of 2 kHz on each side of F_c is added so that the new channel bandwidth is 30kHz. Estimate the range of allowable carrier frequency.

(6 marks)

(c) Determine the reflection coefficients $\{K_m\}$ of the lattice filter corresponding to the FIR filter described by the system function

$$H(z) = 1 - 2.2z^{-1} + 1.92\, z^{-2} - 0.72\, z^{-3}$$

(8 marks)

(d) (i) What are the linear predictors of a time sequence? (3 marks)

(ii) How are values of the autocorrelations of the sequence used to obtain the linear predictors? (3 marks)