Signals

Continuous time or discrete time

Is the signal continuous or sampled in time?

Continuous valued or discrete valued

Can the signal take any value or only discrete values?

Deterministic versus random

Can the 'shape' and the values of the signal be described and analysed by linear system techniques or do the values look like a sequence of random numbers?

Frequency

- Continuous time signals can be characterised by a set of frequency components whose value can be to infinity
- Discrete time signals can be characterised by a limited set of frequencies limited to half the sampling frequency

Frequency in discrete time signals

A discrete-time sinusoidal signal is given by

$x(n) = A \cos(\omega n + \theta)$ where

n is an integer variable (the sample number), A is the amplitude, ω is the frequency in radians per sample, θ is a phase offset in radians

The normalised frequency range is from $-\pi$ to $+\pi$ radians

A continuous sinusoid of 2 kHz sampled at 8000 samples per second has a normalised (wrt sampling frequency) frequency of

$$\frac{2000}{8000} \cdot 2\pi = \pi/2 \text{ radians per sample}$$

Discrete time sinusoids whose frequencies are separated by an integer multiple of 2π are identical.

The highest rate of oscillation in a discrete time sinusoid is at

$\omega = \pi$ (or $\omega = -\pi$)

Frequency in discrete time signals

A discrete time sinusoid is periodic only if its frequency f is a rational number

$f_0 = k/N$ where N is usually the fundamental period and k is an integer

A set of harmonically related complex exponentials is given by

$$s_k(n) = e^{j2\pi k f_0 n} \quad k = 0, \pm 1, \pm 2, ..$$

Using $f_0 = 1/N$ as the fundamental frequency

$$s_{k+N}(n) = e^{j2\pi n(k+N)/N} = e^{j2\pi n}. s_k(n) = s_k(n)$$

This means that there are only N distinct periodic complex exponentials in the set.

Aliasing

A continuous time signal that has a frequency component value higher than half the sampling frequency is distorted when sampled.

The frequency component is transformed (aliased) into a lower frequency component altering (distorting) the original waveform.

To avoid frequency aliasing every digital system **must be preceded by a low pass analog filter** with cutoff at half the intended sampling frequency of the analog-to-digital converter.

Quantising

Since the continuous value is (normally) discretised there is an error within the discrete system.

Setting a discrete step of Δ the quantisation error is within the range $-\Delta/2$ to $\Delta/2$

The mean square error power is $P_q = \frac{\Delta^2}{12}$

Assuming a range $\pm A$ and b bits in the word then $\Delta = 2A/2^b$

Hence

$$P_q = \frac{A^2/3}{2^{2b}}$$

The average signal power is $A^2/2$. Therefore the signal-to-quantisation noise ratio (SQNR) is given by

$$\frac{P_S}{P_q} = \frac{3}{2} \cdot 2^{2b}$$

The SQNR increases approximately 6dB for every bit added to the word length

Discrete signals

Impulse (unit sample) $\delta(n) = 1 \quad n = 0$

$= 0 \quad$ otherwise

Unit step signal $u(n) = 1 \quad n \leftrightarrows 0$

$= 0 \quad n < 0$

Unit ramp signal $r(n) = n \quad n \leftrightarrows 0$

$= 0 \quad n < 0$

Exponential $x(n) = a^n$

Classification of discrete signals

Energy signals and power signals E = $\sum_{n=-\infty}^{n=\infty} |x(n)|^2$

Many signals having infinite energy have a finite average power
Given by

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{n=N} |x(n)|^2$$

If E is finite P = 0. But if E is infinite average power may be
Finite or infinite. If P is finite and non-zero it is called a
Power signal

Periodic signals are given by $x(n+N) = x(n)$.
If the energy over one period is finite the signal is a power signal.
However the energy of the periodic signal is infinite.

Symmetric $x(-n) = x(n)$

Antisymmetric $x(-n) = - x(n)$

Signals are shifted in time by replacing n with n –k
Given $x(n)$, $x(n-2)$ is $x(n)$ delayed by two units in time.
$x(n+3)$ is $x(n)$ advanced by 3 units of time

Operations on Sequences

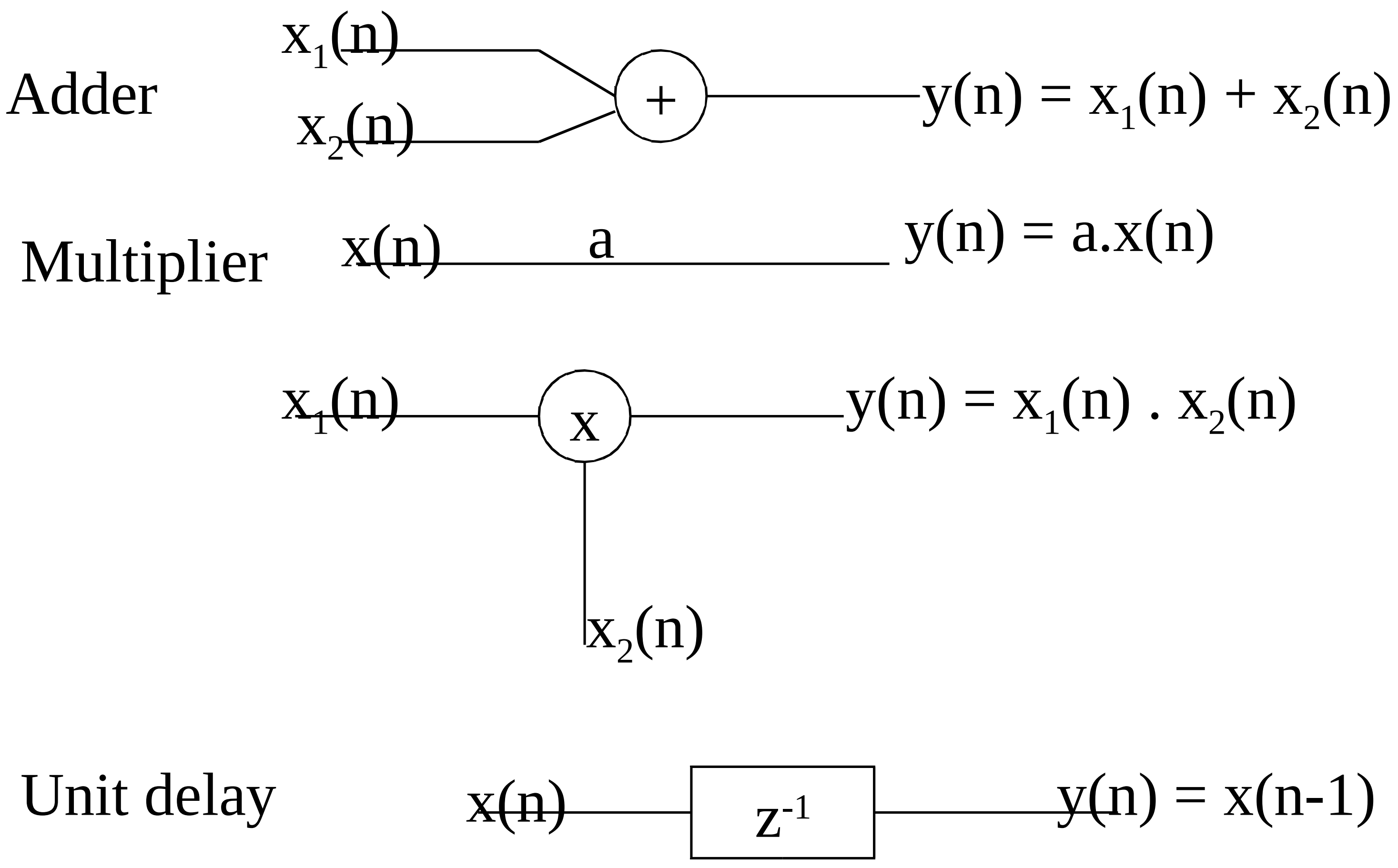
Addition: The sum of two sequences $x_1(n)$ and $x_2(n)$ is

$$y(n) = x_1(n) + x_2(n) \quad \text{for all } n$$

Multiplication $y(n) = x_1(n) \,.\, x_2(n)$ for all n

Scaling $y(n) = A.\, x(n)$

Block diagram representations



Classification of sequences

Time invariant vs time variant

Linear vs non linear systems

Causal vs non causal systems

Stable vs unstable systems

For most of our analysis we assume that the sequences we are working with belong to the class of linear, time-invariant (LTI) systems.

LTI sequences

A sequence x(n) may be represented in terms of impulse responses by

$$x(n) = \sum_{k=-\infty}^{k=\infty} x(k).\delta(n-k)$$

Generalising to an arbitrary transfer function h(n), the response y(n)
For an input x(n) is given by

$$y(n) = \sum_{k=-\infty}^{k=\infty} x(k).h(n-k) \quad = \quad \sum_{k=-\infty}^{k=\infty} h(k).x(n-k)$$

y(n) = x(n) * h(n) = h(n) * x(n)

LTI systems

An LTI system can have

A Finite Impulse Response (FIR) or an Infinite Impulse Response (IIR) Systems whose output depend only on present and past inputs are FIR. Systems who depend also on past outputs are IIR. An FIR system is also a nonrecursive system. A system that depends on past outputs is a recursive system.

In general

$$y(n) = -\sum_{k=1}^{k=N} a_k . y(n-k) + \sum_{k=1}^{k=M} b_k . x(n-k)$$

If the a_k 's are 0 the system is an FIR non-recursive system.

LTI system Properties

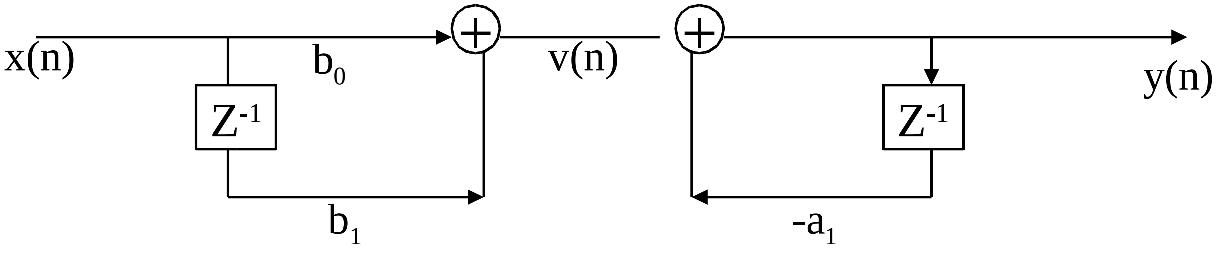
Commutative $x(n)*h(n) = h(n)*x(n)$

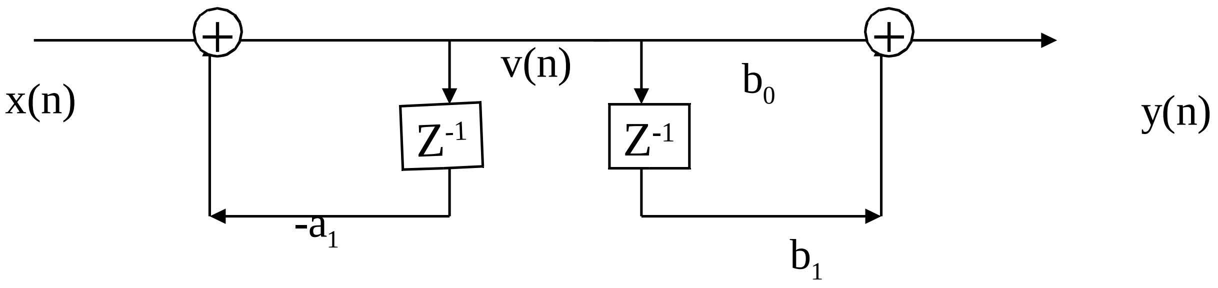
Associative $[x(n)*(h_1(n)]*h_2(n) = x(n)* [h_1(n)]*h_2(n)]$

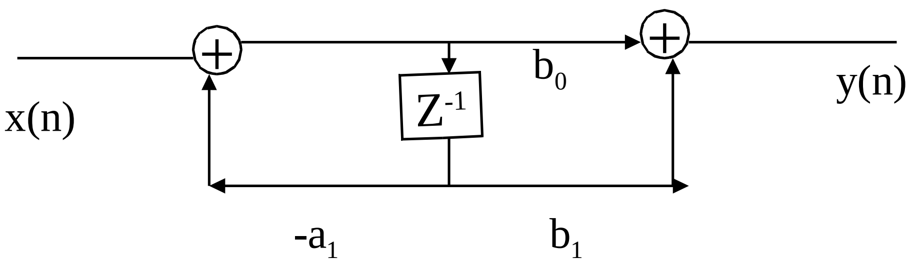
Distributive $x(n)*[h_1(n)+h_2(n)] = x(n)h_1(n) + x(n)*h_2(n)$

Implementation of Discrete Time Systems

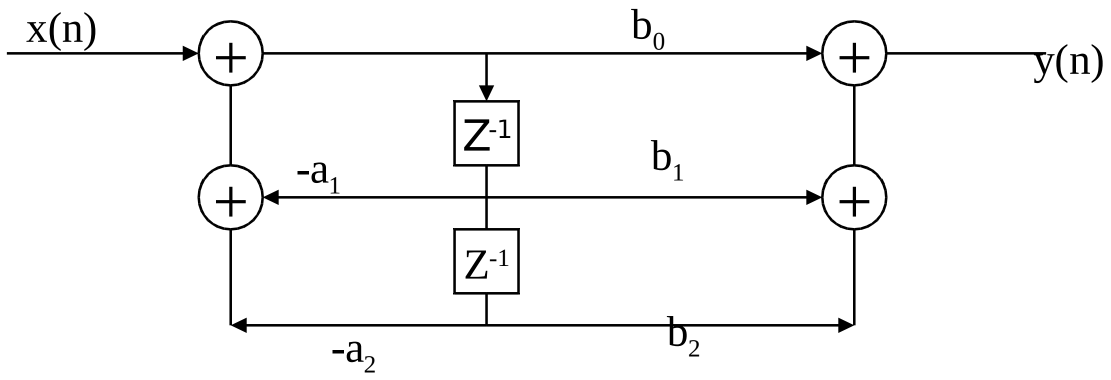
$y(n) = -a_1 y(n-1) + b_0 x(n) + b_1 x(n-1)$







Second Order system Structures



Z-Transform

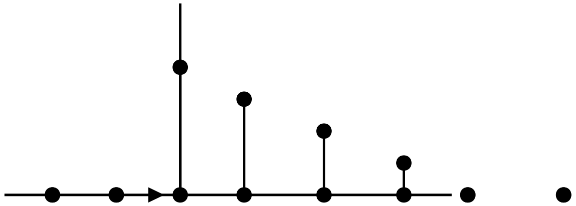
$$X(z) \equiv \sum_{n=-\infty}^{n=\infty} x(n) z^{-n}$$

Since the z-transform is a power series, it exists only for values of z for which the series converges. Hence every z-transform ha Region Of Convergence

For an FIR system the ROC is the entire z-plane with possibly the Exception of z=0 and/or z= infinity

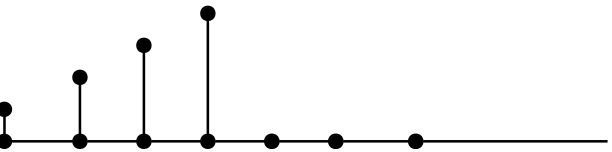
Characteristic ROC for Finite Duration Signals

causal



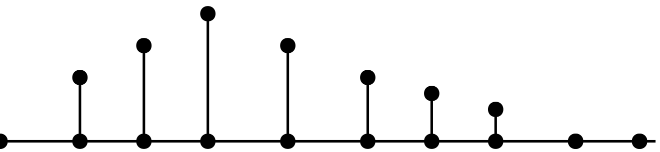
Entire z-plane except z=0

anticausal



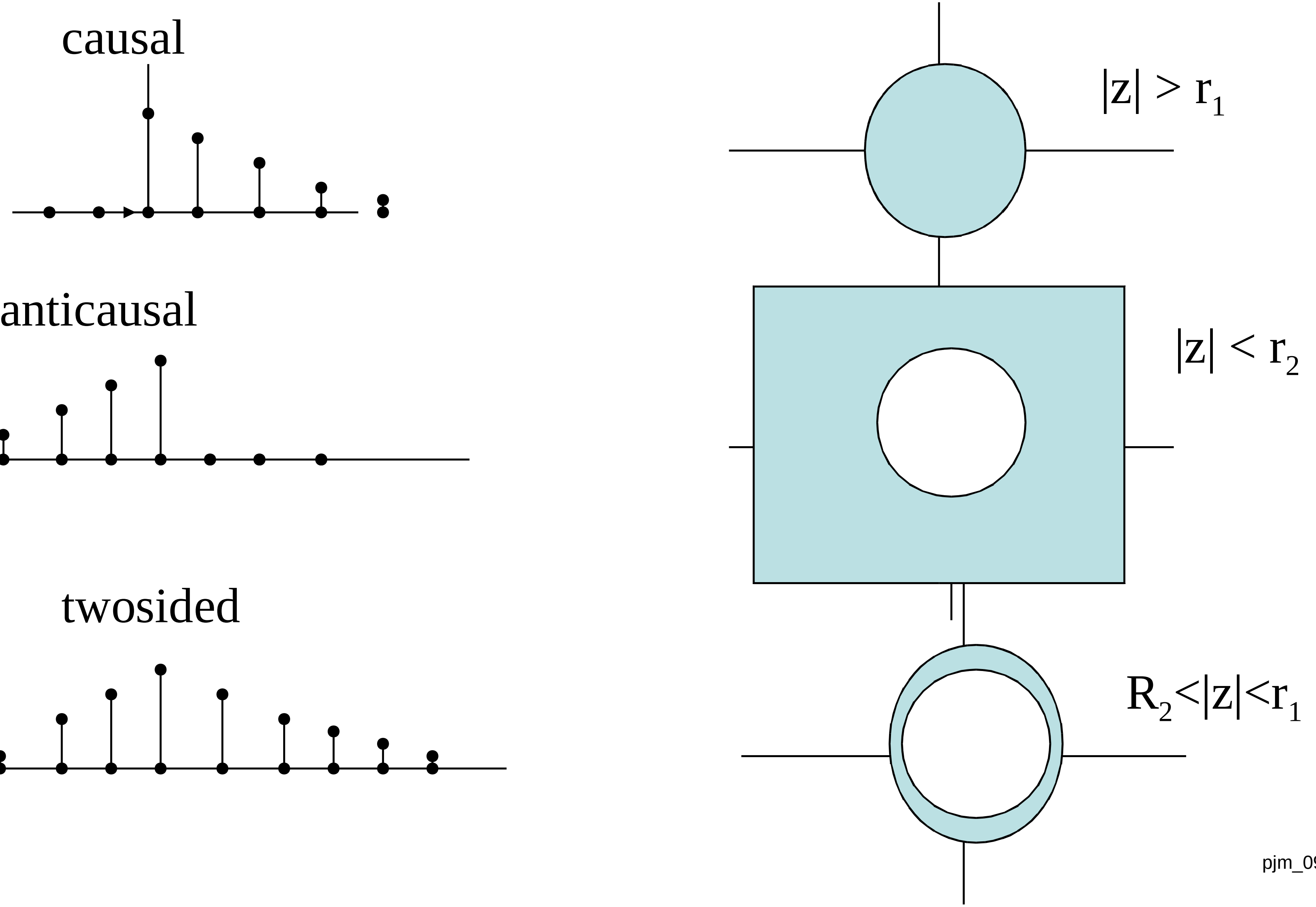
Entire z-plane except x = infinity

Two-sided



Entire z-plane except z=0 and
z = infinity

Characteristic ROC for Infinite Duration Signals



One sided z-transform

This is given by $X(z) \equiv \sum_{n=0}^{n=\infty} x(n) z^{-n}$

It does not contain information of x(n) n<0. It is unique for causal signals,. The ROC must be the exterior of a circle which can extend To z=0. Hence the ROC is implicit.
Most of the properties of the two sided z-transform carry over into The one sided z-transform

Properties of the z-transform

Linearity if $x(n) = a_1x_1(n) + a_2x_2(n)$ then $X(z) = a_1X_1(z) + a_2X_2(z)$

Time shifting $x(n-k) = z^{-k} X(z)$

Scaling $a^n x(n) = X(a^{-1} z)$

Time reversal $x(-n) = X(z^{-1})$

Differentiation in z-domain $nx(n) = -z \frac{dX(z)}{dz}$

Convolution if $x(n)=x_1(n) * x_2(n)$ then $X(z)= X_1(z) . X_2(z)$

Multiplication if $x(n) = x_1(n)\, x_2(n)$ then $X(z) = X_1(z) * X_2(z)$

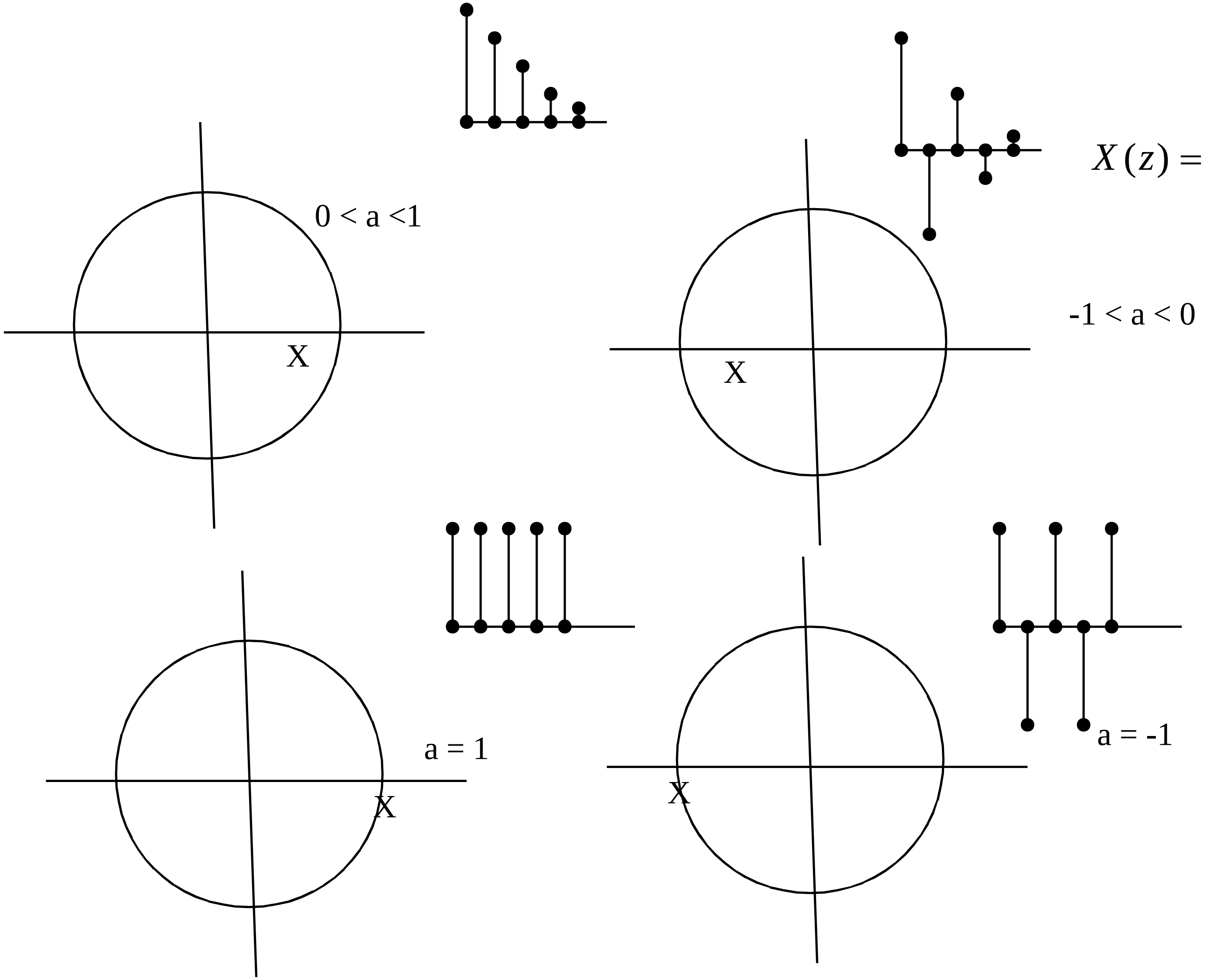
Poles and Zeroes

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=1}^{N} 1 - a_k z^{-k}}$$

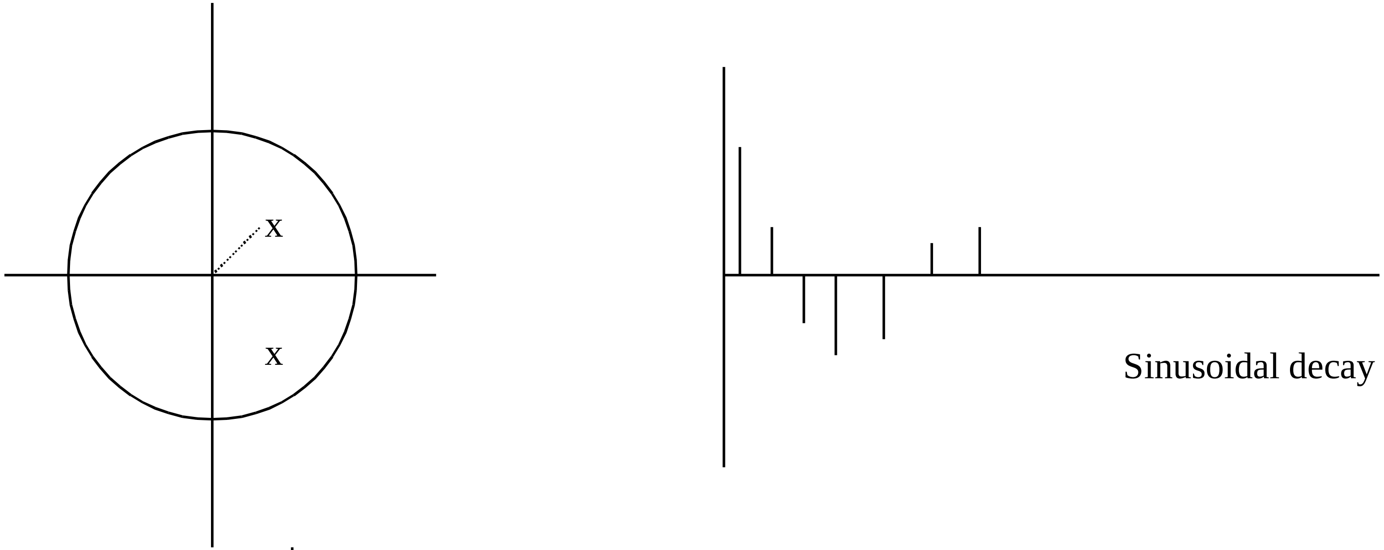
In general the numerator power series has as roots the zeroes of X(z) while the denominator roots are the poles of X(z). Two important special forms are when all a_k are zero. In this case the solution is an all zero system and has a finite duration impulse response.

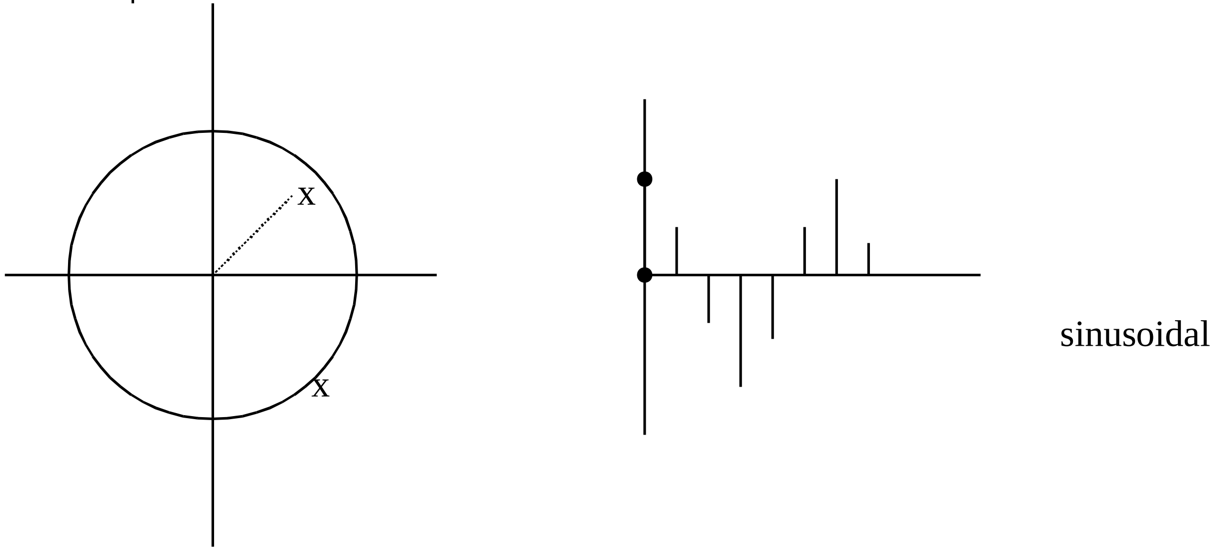
Pole Location and Time Domain Behaviour

$$X(z) = \frac{1}{1 - az^{-1}}$$

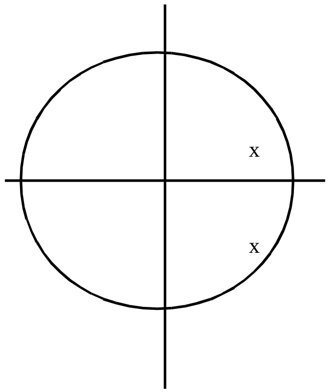


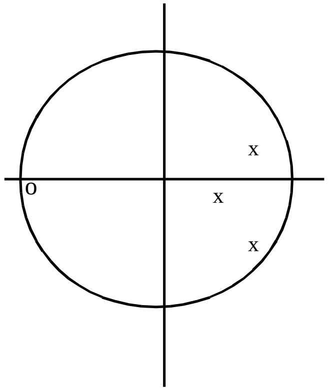
Complex Conjugate Poles



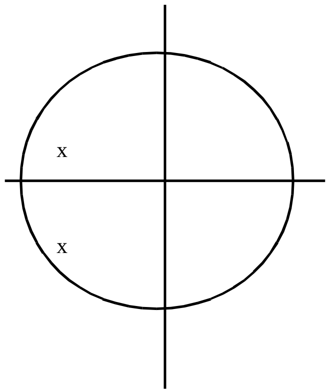


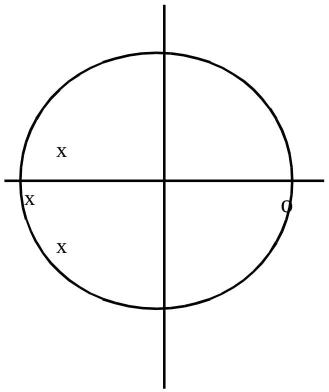
Pole Location and Frequency Domain Behaviour





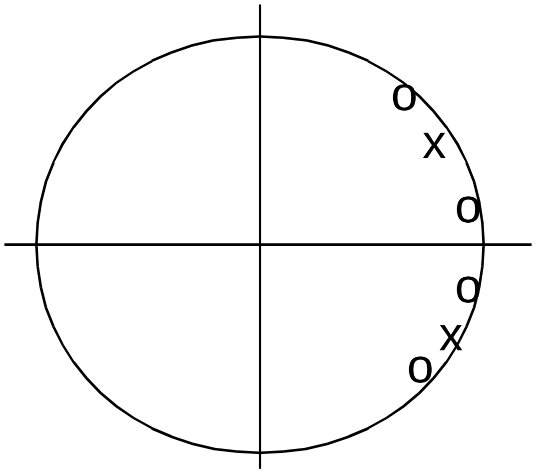
Lowpass



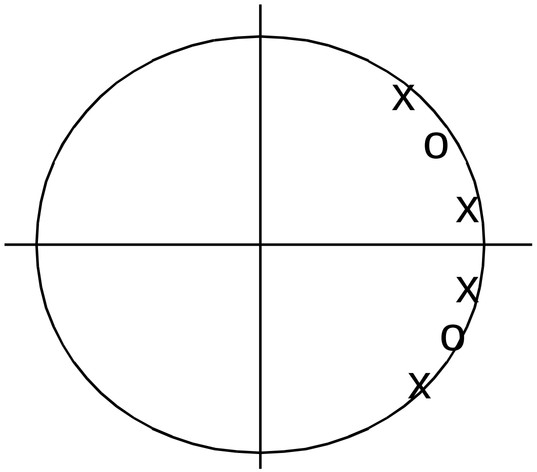


highpass

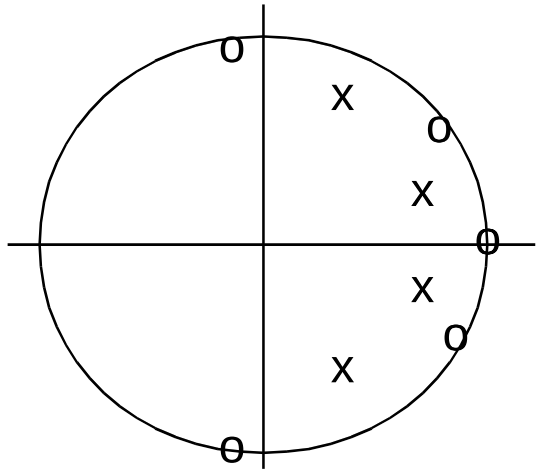
Pole and Zero Location and Frequency Domain behaviour



bandpass

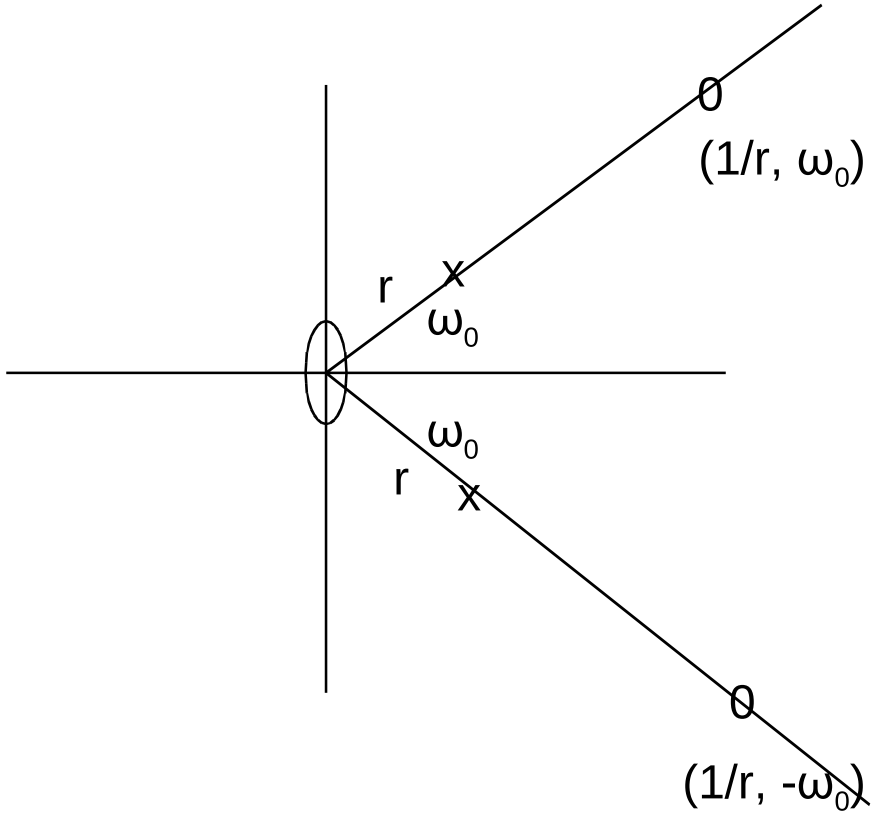


bandstop



Notch filter

Pole and Zero location of filters – All pass filter



$$H(z) = \frac{(r^2 - 2r\cos\omega_0 z^{-1} + z^{-2})}{(1 - 2r\cos\omega_0 z^{-1} + r^2 z^{-2})}$$

Minimum and Maximum Phase

An FIR system with M zeros can be characterised by

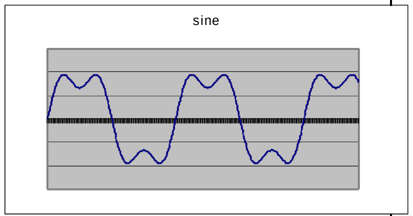
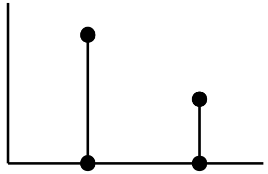
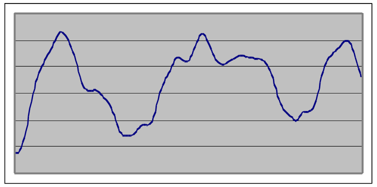
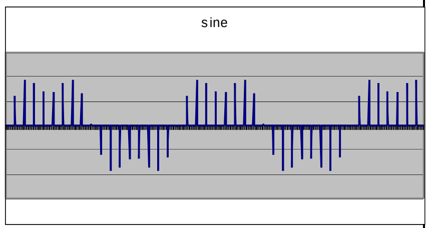
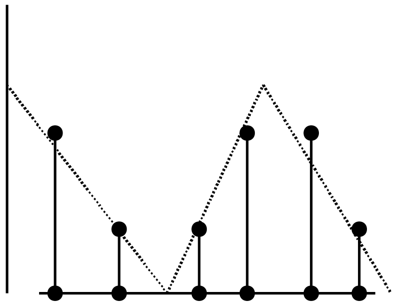
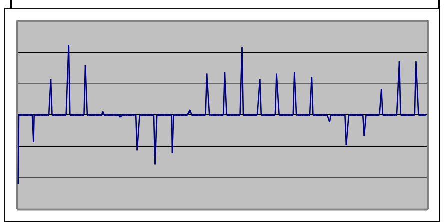
$$H(w) = b_0(1 - z_1 e^{-jw})(1 - z_2 e^{-jw}) \cdots (1 - z_M e^{-jw})$$

Where z_i denote the zeros. If all the zeros are inside the unit circle, each term Corresponding to a real valued zero undergoes a net phase change of zero between $\omega=0$ and $\omega=\pi$. Similarly each pair of complex conjugate zeroes will undergo a net phase change of zero. System called MINIMUM PHASE

When all the zeroes are outside the unit circle. A real valued zero contributes a net Phase change of π radians and a complex conjugate pair a net phase change of 2π radians over the range $\omega=0$ to $\omega=\pi$, which is the largest possible phase Change. System called MAXIMUM PHASE.

Magnitude response remains the same if one zero at z_k inside unit circle is Reflected outside the unit circle at $1/z_k$. But phase change alters

Sampling in Time and Frequency

Continuous Periodic		Line Spectrum	
Continuous Aperiodic		Continuous Spectrum	
Sampled Periodic		Line Spectrum Repetitive	
Sampled Aperiodic		Continuous Spectrum Repetitive	

Discrete Fourier Transform

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi n \frac{k}{N}}$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi k \frac{n}{N}}$$

This result requires that the time record length L is less or equal to N, and the Frequency spectrum accuracy of $2\pi/N$ requires N non-zero time samples.

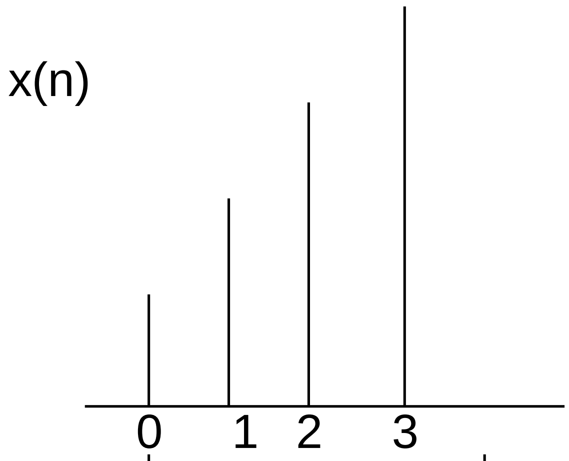
Properties of the DFT

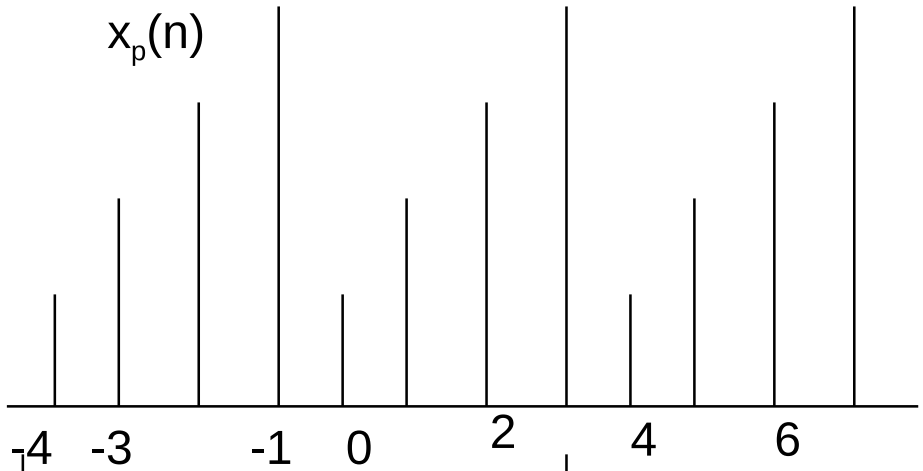
The most important property relates to circular shift. This property comes from the fact that the time record of an N-point DFT is a periodic sequence $x_p(n)$ of Period N.

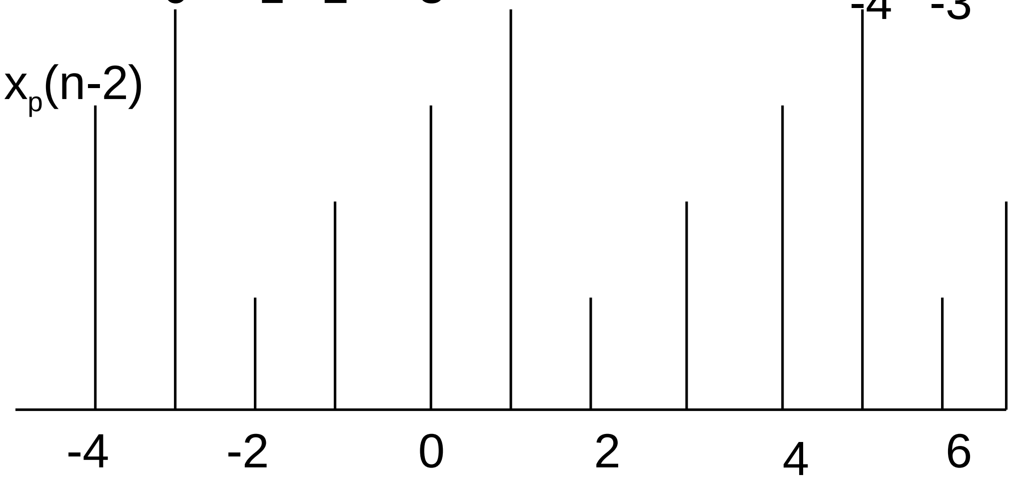
Shifting the periodic sequence $x_p(n)$ by k units to the right is equivalent to

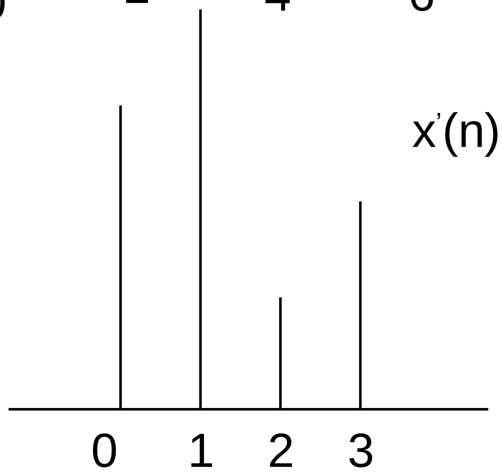
$$x'(n) = x_p(n-k) = x(n-k, \text{modulo } N)$$

Circular Shift

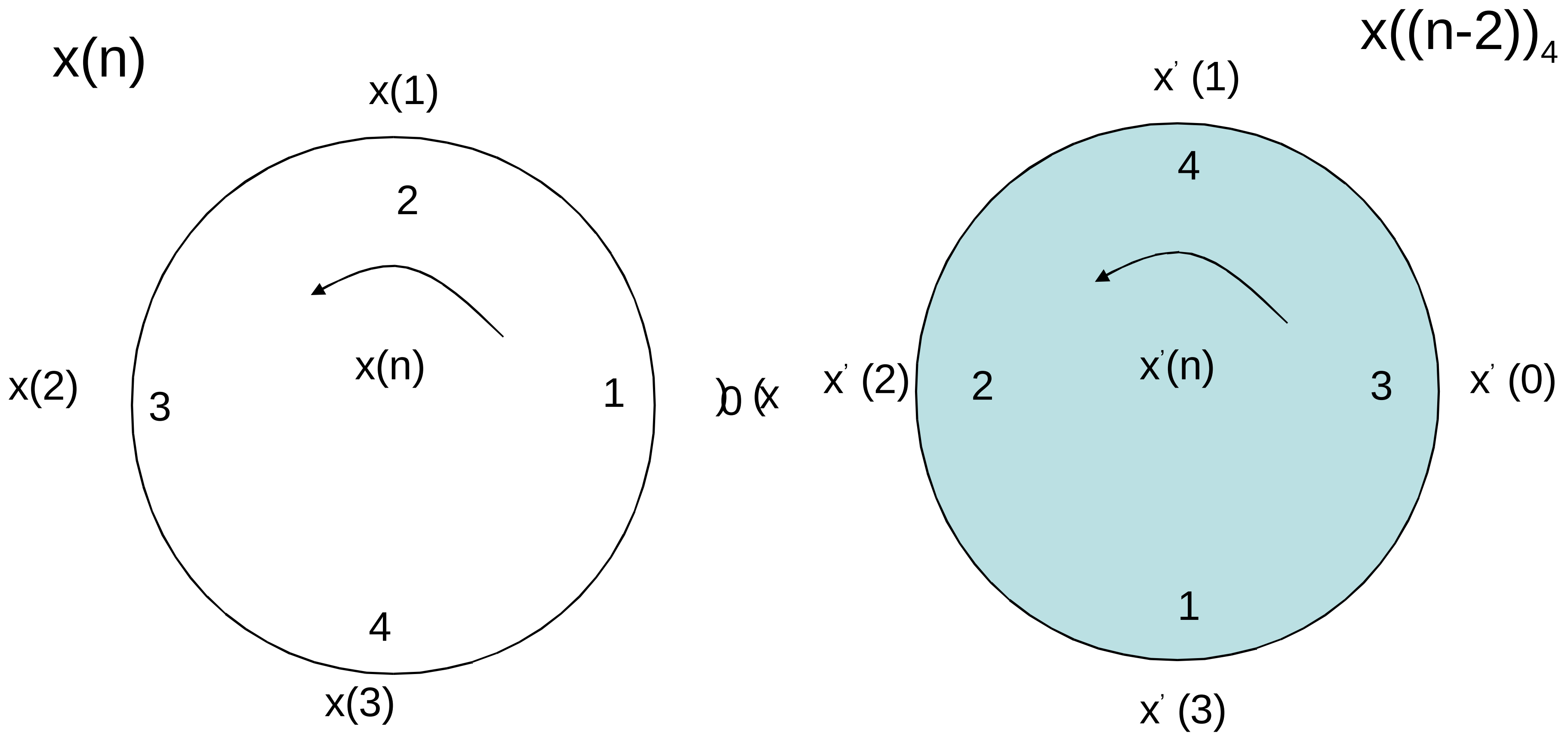








Circular Shift



Convolution with DFT's

$x_1(n) \longrightarrow X_1(k)$

$x_2(n) \longrightarrow X_2(k)$

$x_1(n) * x_2(n) \longrightarrow X_1(k).\ X_2(k)$

Multiplication of two DFT's implies convolution of two periodic time sequences.

This results in CIRCULAR CONVOLUTION

Circular Convolution

$$X_3(k) = X_1(k).X_2(k)$$

$$x_3(m) = \sum_{n=0}^{N-1} x_1(n).x_2((m-n))_N \cdots m = 0,1,\cdots N-1$$

This is not linear convolution. Note that in this case $x_1(n)$ is of length N, $x_2(n)$ is also of length N, and the result $x_3(n)$ is also of length N.
In linear convolution the result of convolving a sequence of length N_1 with one of length N_2, is an output sequence of length $N_1 + N_2 – 1$.

Linear Convolution using the DFT

For a signal of length N_1 passed through a filter of length N_2 the linear convolution results in $N_1 + N_2 – 1$.
Therefore EACH of the two time signals are brought to a length of at least $N_1 + N_2 – 1$by padding zeroes after the non-zero samples.
Since both signals are of length $N_1 + N_2 – 1$, the result of the circular convolution Has also $N_1 + N_2 – 1$points.
This circular convolution is however equivalent to a linear convolution

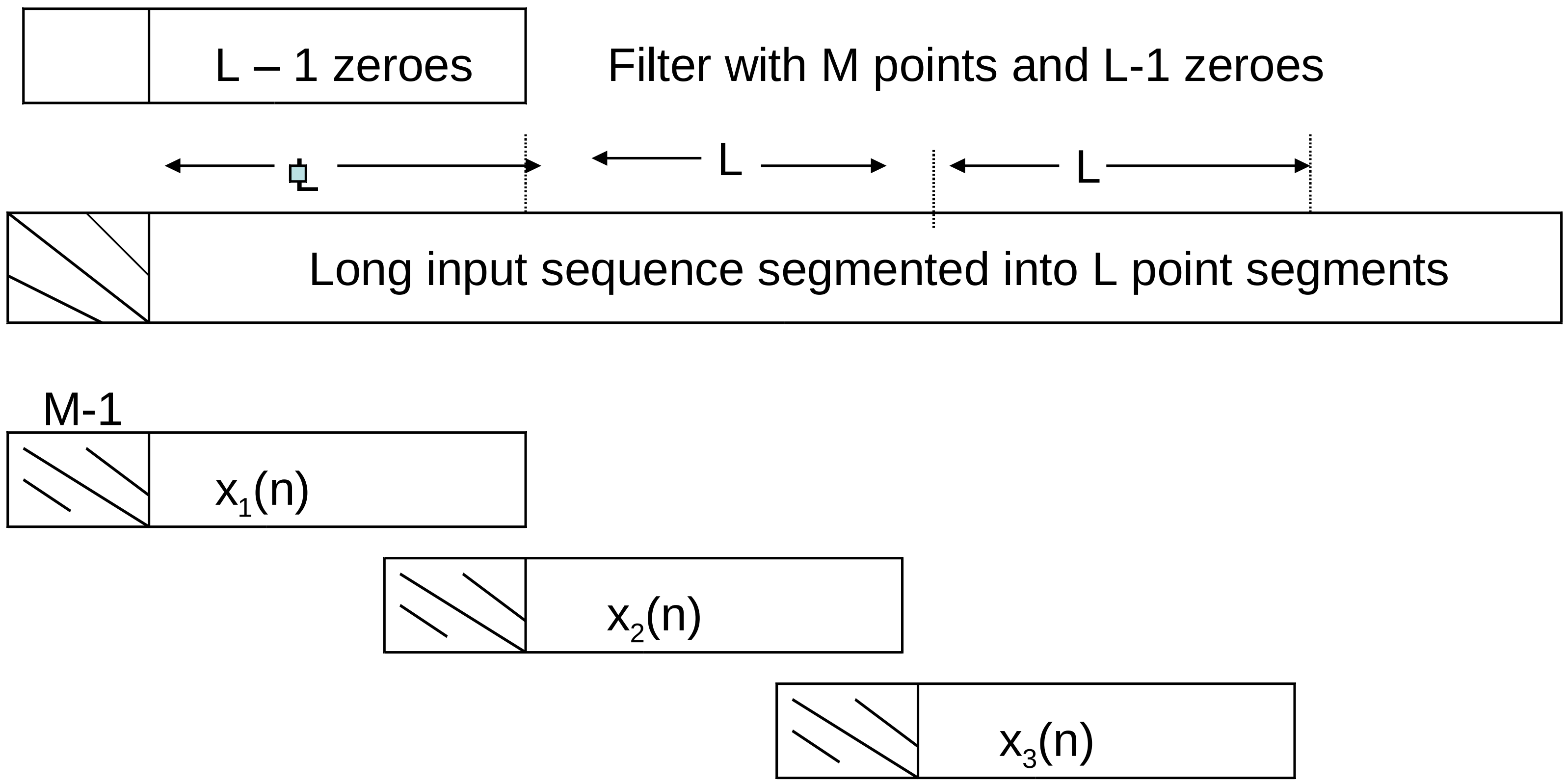
Long input sequences

When the input sequence to be filtered is very long, it is necessary to break the Signal into segments, do the processing, and then reunite again the segments.
The overall effect must however be the same as if the signal filtered is continuous.
This requires consideration both of valid samples in the output, as well as of the time For processing with respect to a real time application.
Two methods are used

OVERLAP SAVE

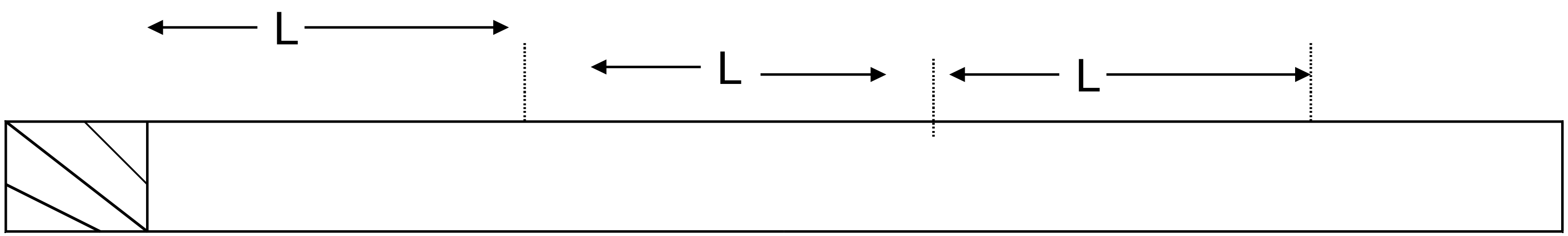
OVERLAP ADD

Overlap and Save Method

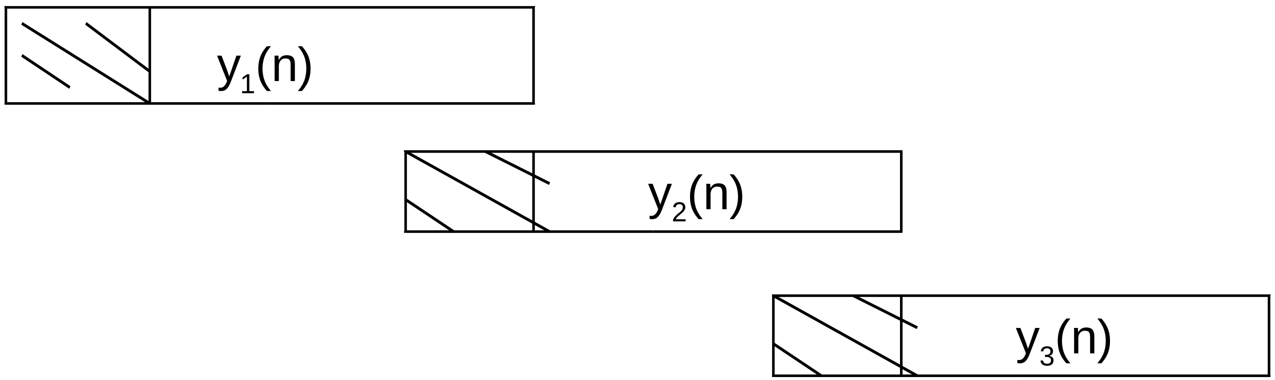


In this case the first segment has M-1 zeroes pre-added. Each new segment makes use of M-1 samples from the previous segment, so that every segment is L+M-1 samples long as required for linear convolution.

Overlap and Save method

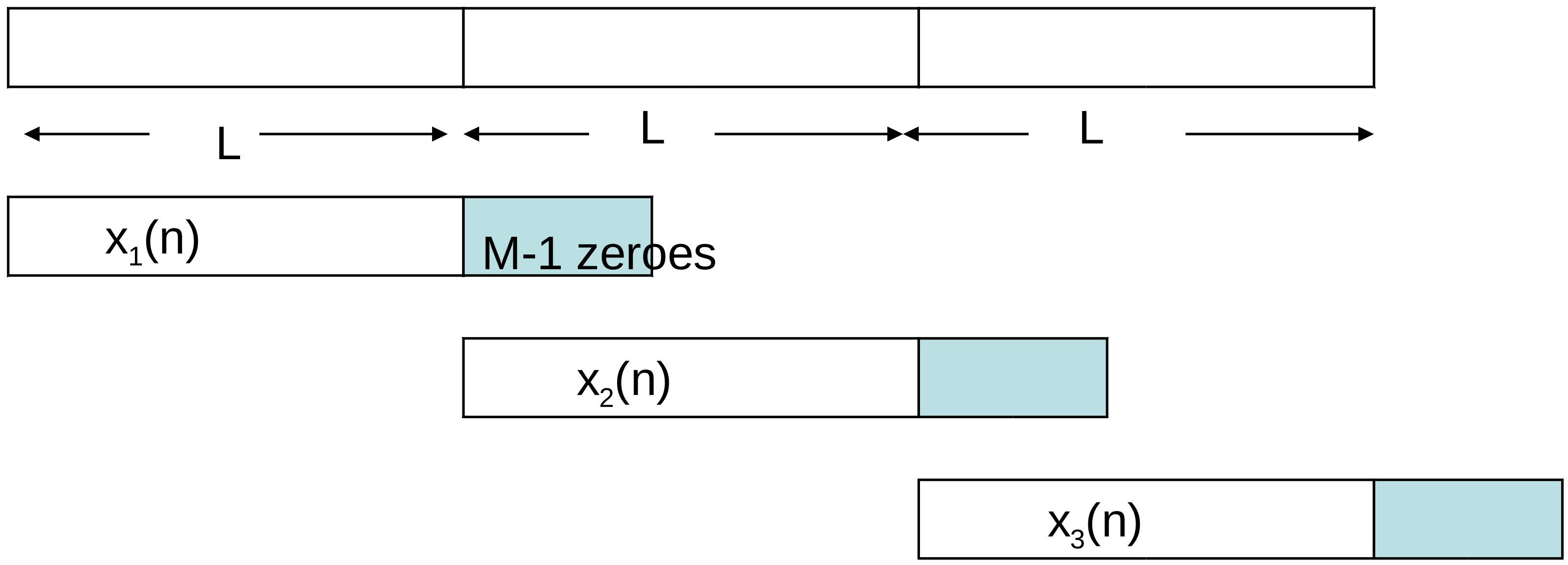


Discard first M-1 output samples
from each output segment



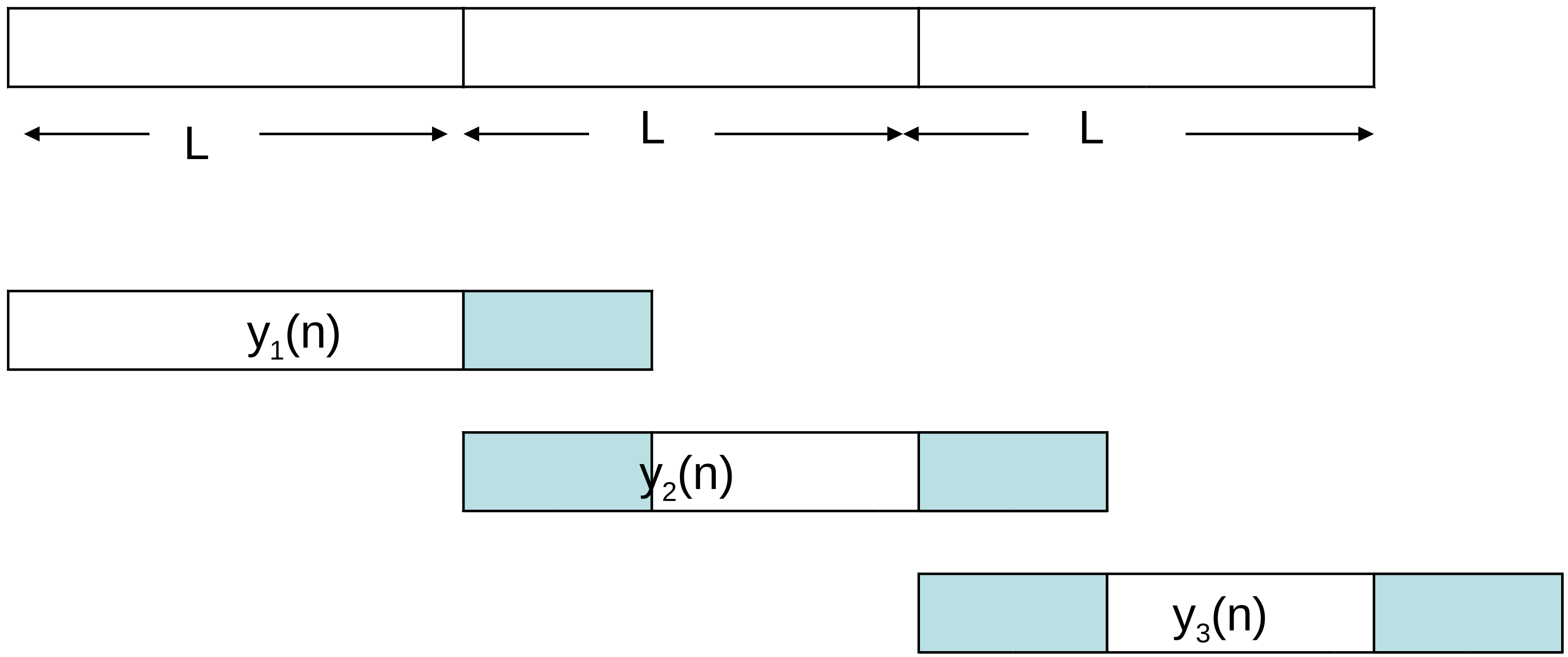
When the valid samples from each segment are abutted the result is a linear convolution of the long sequence with the filter

Overlap and Add method



In this case each segment has M-1 zeroes appended to make up the necessary length of L+M-1 for linear convolution

Overlap and Add method



In this case the result of the circular convolution is all valid for the resultant linear convolution. The end (M-1 samples) part of a segment is added to the front part of the subsequent segment to give the proper result for that part

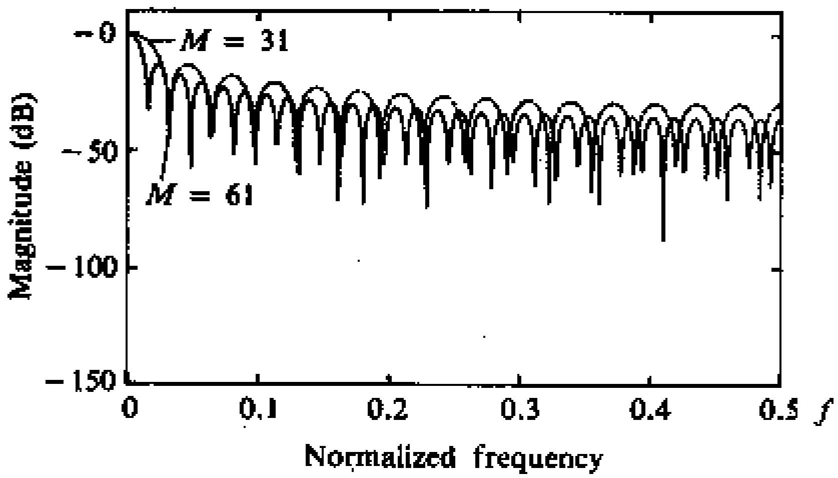
Number of samples in the time sequence and its frequency DFT response

In the analogue domain a periodic waveform is assumed to have a line frequency spectrum assuming the time waveform is infinite in length. A discrete sequence x(n) having L non zero samples can be considered as an infinite sequence x(n) multiplied by

r(n)	1	$0 < n < L$
	0	otherwise

If x(n) has a frequency transform X(k) and r(n) has a frequency transform R(k), then the frequency transform x(n). r(n) is given by X(k)*R(k) where * denotes convolution

Frequency Transform of a Rectangular Window



Note L is M in figure

$$R(\omega) = \sum_{n=0}^{L-1} e^{-j\omega n} \quad = \quad \frac{(1 - e^{-jwL})}{1 - e^{-j\omega}} \quad = \quad \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}$$

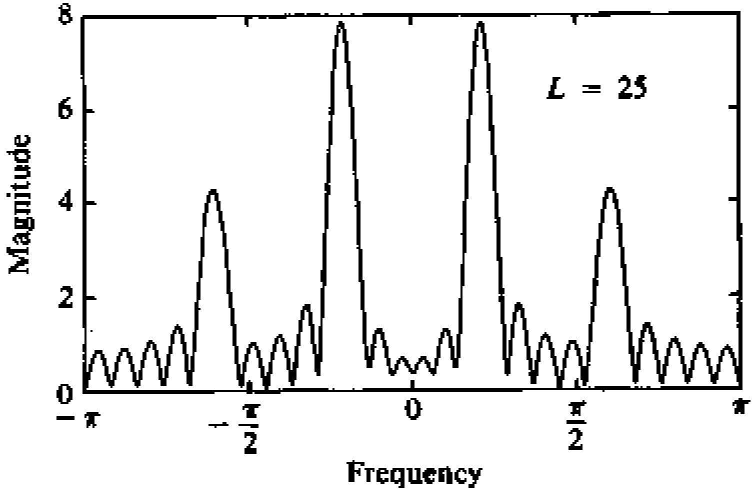
This has the first zero at $\omega L/2 = \pi$ or $\omega = 2\pi/L$

Spectral Leakage

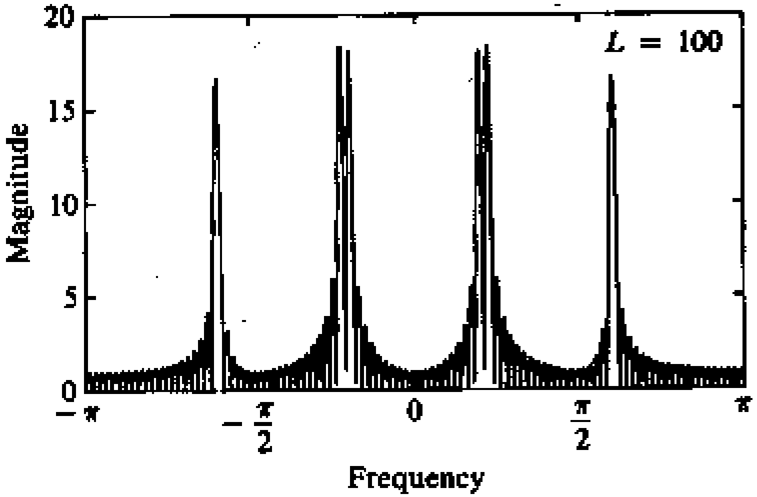
Windowing reduces spectral resolution.

The spectrum $W(\omega)$ of the rectangular window sequence has its first zero crossing at $\omega = 2\pi/L$. Now if $|\omega_1 - \omega_2| < 2\pi/L$, the two window functions $W(\omega - \omega_1)$ and $W(\omega - \omega_2)$ overlap and, as a consequence, the two spectral lines in $x(n)$ are not distinguishable. Only if $(\omega_1 - \omega_2) \geq 2\pi/L$ will we see two separate lobes in the spectrum $\hat{X}(\omega)$. Thus our ability to resolve spectral lines of different frequencies is limited by the window main lobe width. Figure 5.13 illustrates the magnitude spectrum $|\hat{X}(\omega)|$, computed via the DFT, for the sequence

$$x(n) = \cos\omega_0 n + \cos\omega_1 n + \cos\omega_2 n \qquad (5.4.8)$$



Main lobe not sufficient to Distinguish ω1 and ω2 when L is only 25.



Two frequencies ω1 and ω2 are distinguished when L is 100. ($2\pi/L$ is smaller) . The Spectrum has also leaked all over the frequency range.

The convolution X(k) * R(k) results in a smearing of the ideal line frequency spectrum, so that the frequency spectrum is spread and distorted. This is known as spectral leakage.

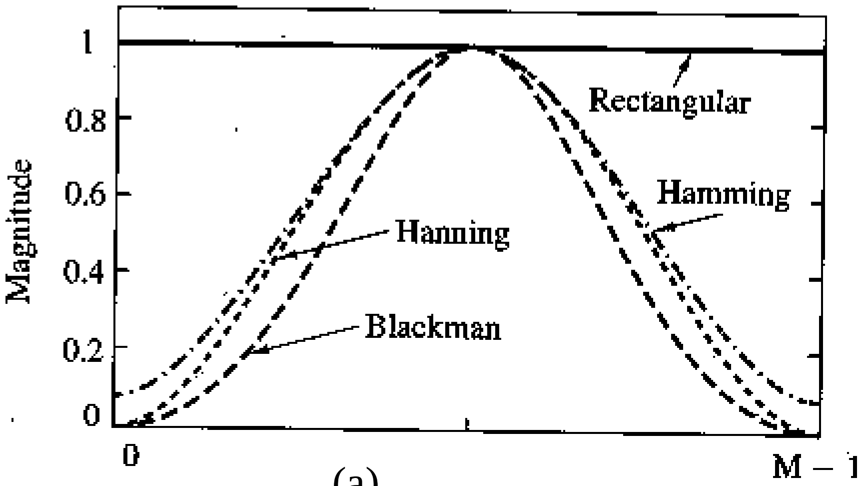
Hamming and Hanning Windows

$h(n) = 0.54 – 0.46 \cos [2\pi n/(M-1)]$ - Hamming function

$h(n) = 0.5[1 \quad - \cos[2\pi n/(M-1)]$ - Hanning function

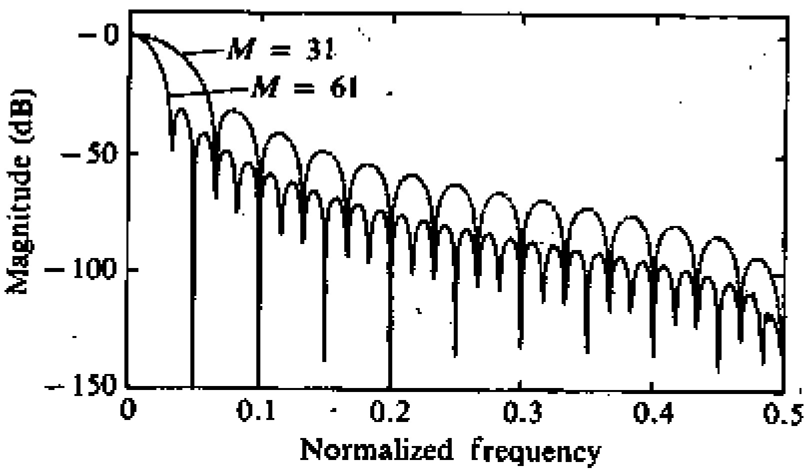
M is the window length in samples

Time and Frequency Response

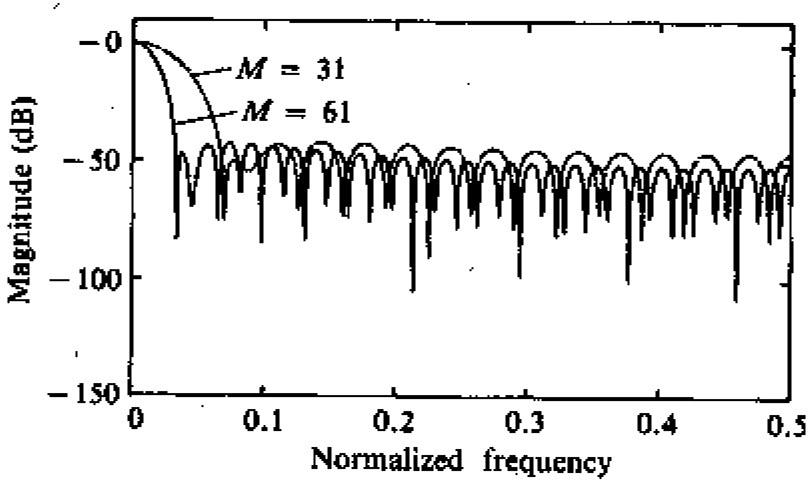


(a)

Time shape (a) and frequency Response of Hanning (b) and Hamming (c)



(b)



(c)

Sampling Requirements of Bandpass Signals

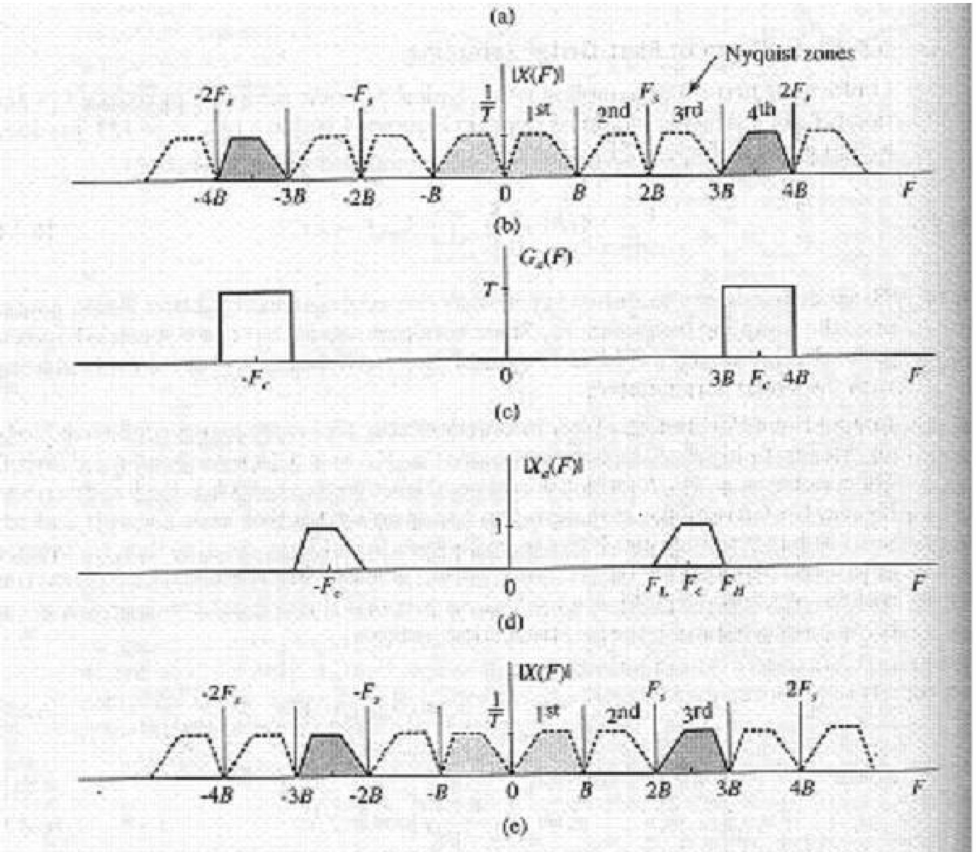


Figure 6.4.1 Illustration of bandpass signal sampling for integer band positioning.

Integer band
Positioning
F_H = mB

Note for m
even the
Inversion of
The baseband
Spectral image

Sampling Requirements for Bandpass Signals

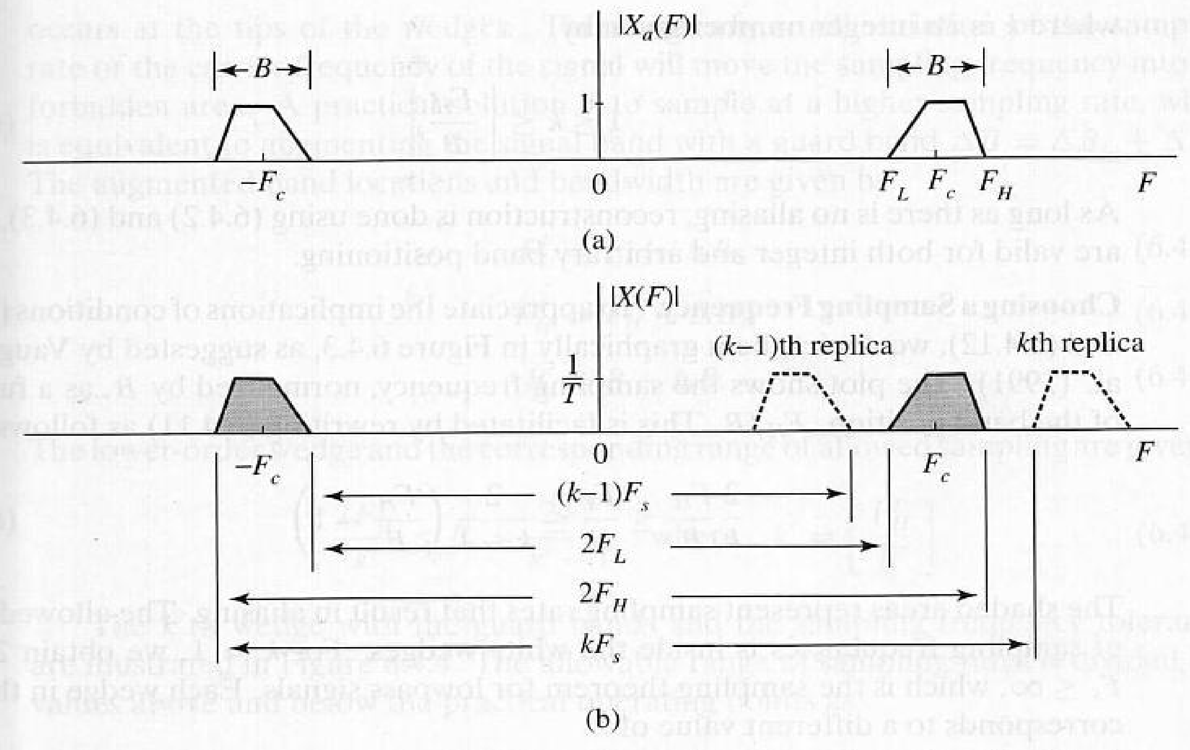


Figure 6.4.2 Illustration of bandpass signal sampling for arbitrary band positioning.

$$\frac{2F_H}{k} \leqslant F_S \leqslant \frac{2F_L}{k+1}$$

Choosing a Sampling Frequency for the Bandpass Signal

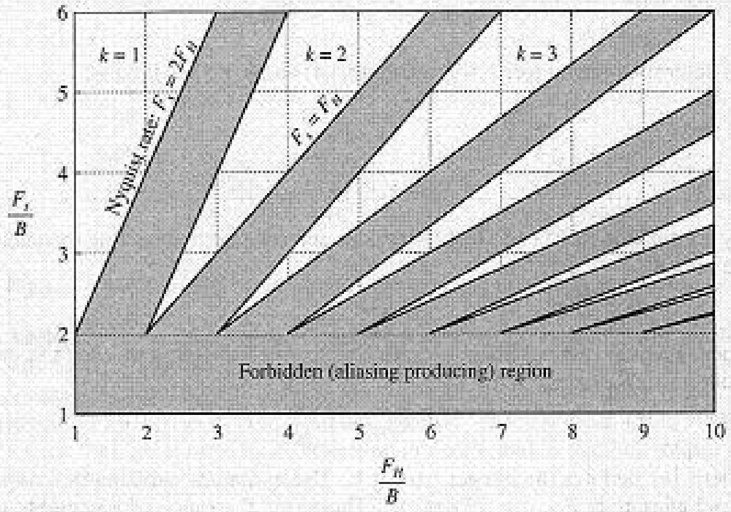


Figure 6.4.3 Allowed (white) and forbidden (shaded) sampling frequency regions for bandpass signals. The minimum sampling frequency $F_s = 2B$, which corresponds to the corners of the alias-free wedges, is possible for integer-positioned bands only.

Choosing a Sampling Frequency. To appreciate the implications of conditions (6.4.11) and (6.4.12), we depict them graphically in Figure 6.4.3, as suggested by Vaughan et al. (1991). The plot shows the sampling frequency, normalized by B, as a function of the band position, F_H/B. This is facilitated by rewriting (6.4.11) as follows:

$$\frac{2}{k}\frac{F_H}{B} \le \frac{F_s}{B} \le \frac{2}{k-1}\left(\frac{F_H}{B} - 1\right) \qquad (6.4.13)$$

The shaded areas represent sampling rates that result in aliasing. The allowed range of sampling frequencies is inside the white wedges. For $k = 1$, we obtain $2F_H \le F_s \le \infty$, which is the sampling theorem for lowpass signals. Each wedge in the plot corresponds to a different value of k.

Choosing a Sampling Frequency for the Bandpass Signal

In practice a guard band is necessary. This results in

$$\frac{2F'_H}{k'} \le F_s \le \frac{2F'_L}{k'-1} \quad \text{where} \quad k' = \left\lfloor \frac{F'_H}{B'} \right\rfloor$$

where

$$F'_L = F_L - \Delta B_L$$

$$F'_H = F_H + \Delta B_H$$

$$B' = B + \Delta B$$

and

$$\Delta F_s = \frac{2F'_L}{k'-1} - \frac{2F'_H}{k'} = \Delta F_{sL} + \Delta F_{sH}$$

From the shaded orthogonal triangles in Figure 6.4.4, we obtain

$$\Delta B_L = \frac{k'-1}{2} \Delta F_{sH}$$

$$\Delta B_H = \frac{k'}{2} \Delta F_{sL}$$

Multirate DSP - Decimation

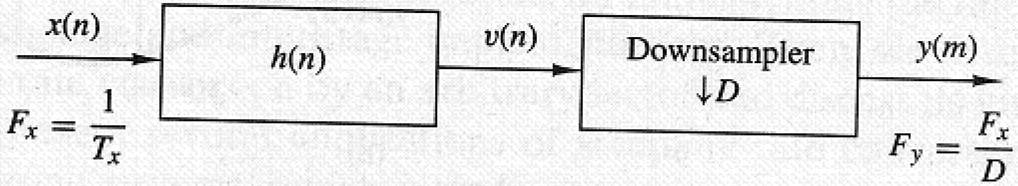


Figure 10.2 Decimation by a factor D.

$$\begin{aligned} y(m) &= v(mD) \\ &= \sum_{k=0}^{\infty} h(k)x(mD-k) \end{aligned}$$

Proakisand Monolakis_10.2

Multirate DSP - Decimation

$$\tilde{v}(n) = \begin{cases} v(n), & n = 0, \pm D, \pm 2D, \ldots \\ 0, & \text{otherwise} \end{cases} \tag{10.2.4}$$

Clearly, $\tilde{v}(n)$ can be viewed as a sequence obtained by multiplying $v(n)$ with a periodic train of impulses $p(n)$, with period D, as illustrated in Fig. 10.3. The discrete Fourier series representation of $p(n)$ is

$$p(n) = \frac{1}{D} \sum_{k=0}^{D-1} e^{j2\pi kn/D} \tag{10.2.5}$$

Hence

$$\tilde{v}(n) = v(n)p(n) \tag{10.2.6}$$

and

$$y(m) = \tilde{v}(mD) = v(mD)p(mD) = v(mD) \tag{10.2.7}$$

$$Y(z) = \sum_{m=-\infty}^{\infty} \tilde{v}(m) z^{-m/D} \qquad = \frac{1}{D} \sum_{k=0}^{D-1} H_D(e^{-j2\pi k/D} z^{1/D}) X(e^{-j2\pi k/D} z^{1/D})$$

Multirate DSP - Decimation

The ideal lowpass filter is given by

$$H_D(\omega) = \begin{cases} 1, & |\omega| \le \pi/D \\ 0, & \text{otherwise} \end{cases}$$

Since the sampling rates are related by the expression

$$F_y = \frac{F_x}{D}$$

$$\omega_y = D\omega_x \qquad (10.2.13)$$

Thus, as expected, the frequency range $0 \le |\omega_x| \le \pi/D$ is stretched into the corresponding frequency range $0 \le |\omega_y| \le \pi$ by the downsampling process.

$$\begin{aligned} Y(\omega_y) &= \frac{1}{D} H_D\left(\frac{\omega_y}{D}\right) X\left(\frac{\omega_y}{D}\right) \\ &= \frac{1}{D} X\left(\frac{\omega_y}{D}\right) \end{aligned}$$

Multirate DSP - Decimation

$X(\omega_x)$

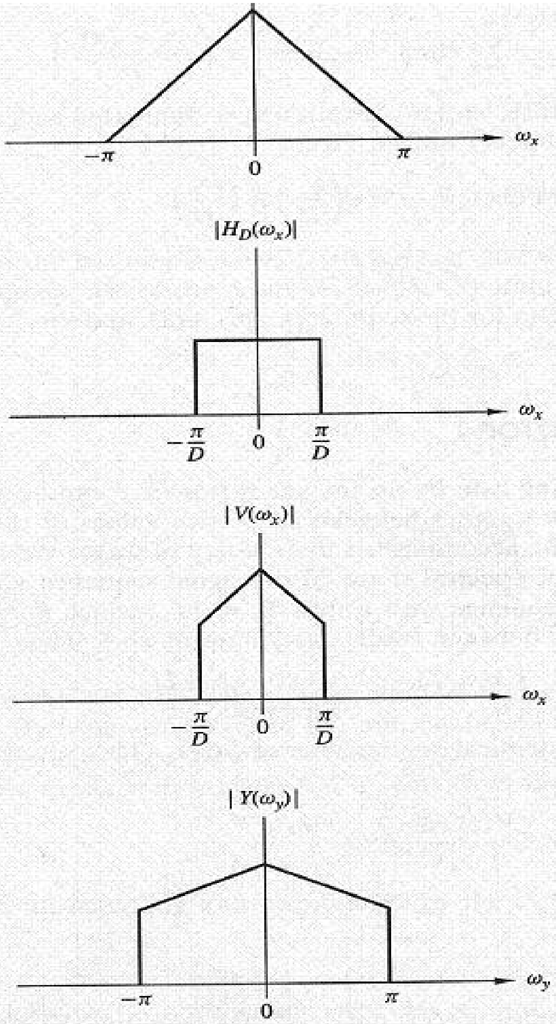


Figure 10.4 Spectra of signals in the decimation of $x(n)$ by a factor D.

Proakisand Monolakis_10.4

Multirate DSP - Interpolation

Let $v(m)$ denote a sequence with a rate $F_y = IF_x$, which is obtained from $x(n)$ by adding $I-1$ zeros between successive values of $x(n)$. Thus

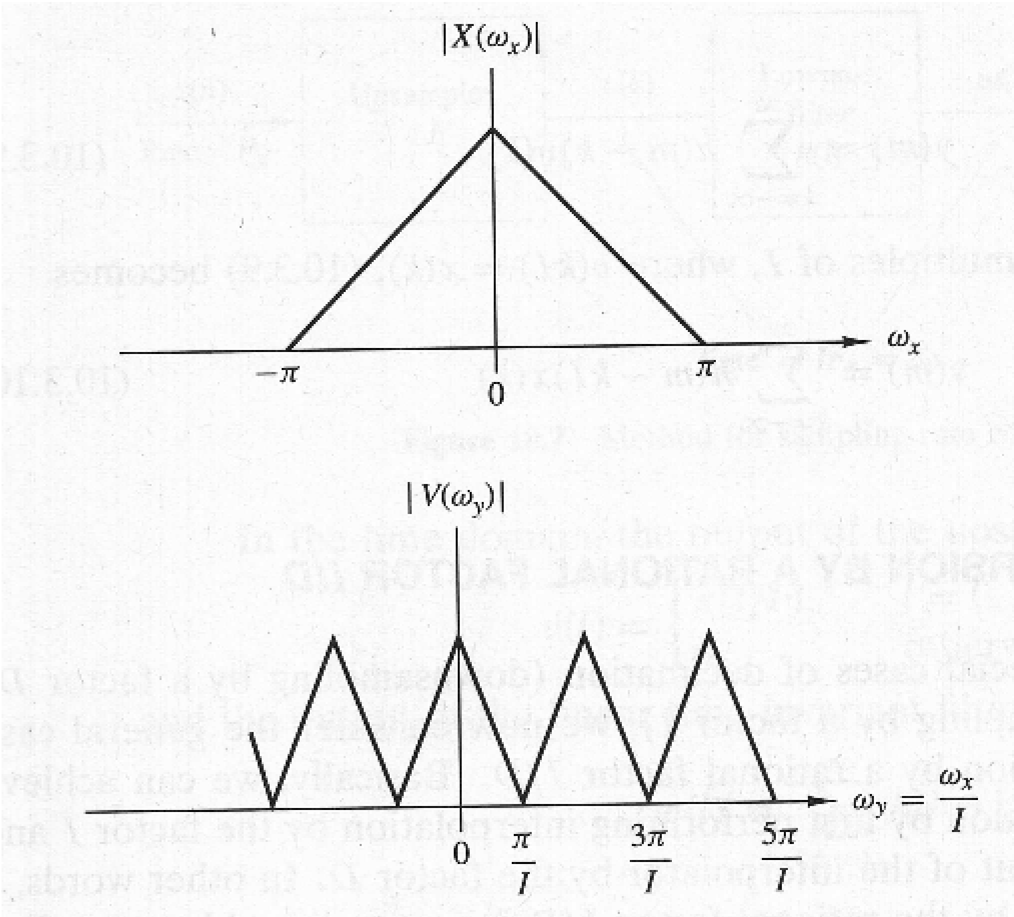
$$v(m) = \begin{cases} x(m/I), & m = 0, \pm I, \pm 2I, \ldots \\ 0, & \text{otherwise} \end{cases} \tag{10.3.1}$$

and its sampling rate is identical to the rate of $y(m)$. This sequence has a z-transform

$$\begin{aligned} V(z) &= \sum_{m=-\infty}^{\infty} v(m) z^{-m} \\ &= \sum_{m=-\infty}^{\infty} x(m) z^{-mI} \\ &= X(z^I) \end{aligned} \tag{10.3.2}$$

$$V(\omega_y) = X(\omega_y I)$$

Multirate DSP - Interpolation



Proakisand Monolakis_10.5

Multirate DSP - Interpolation

A lowpass filter is then used given by

$$H_I(\omega_y) = \begin{cases} C, & 0 \le |\omega_y| \le \pi/I \\ 0, & \text{otherwise} \end{cases} \qquad (10.3.5)$$

where C is a scale factor required to properly normalize the output sequence $y(m)$. Consequently, the output spectrum is

$$Y(\omega_y) = \begin{cases} CX(\omega_y I), & 0 \le |\omega_y| \le \pi/I \\ 0, & \text{otherwise} \end{cases} \qquad (10.3.6)$$

Multirate DSP – Conversion by I/D

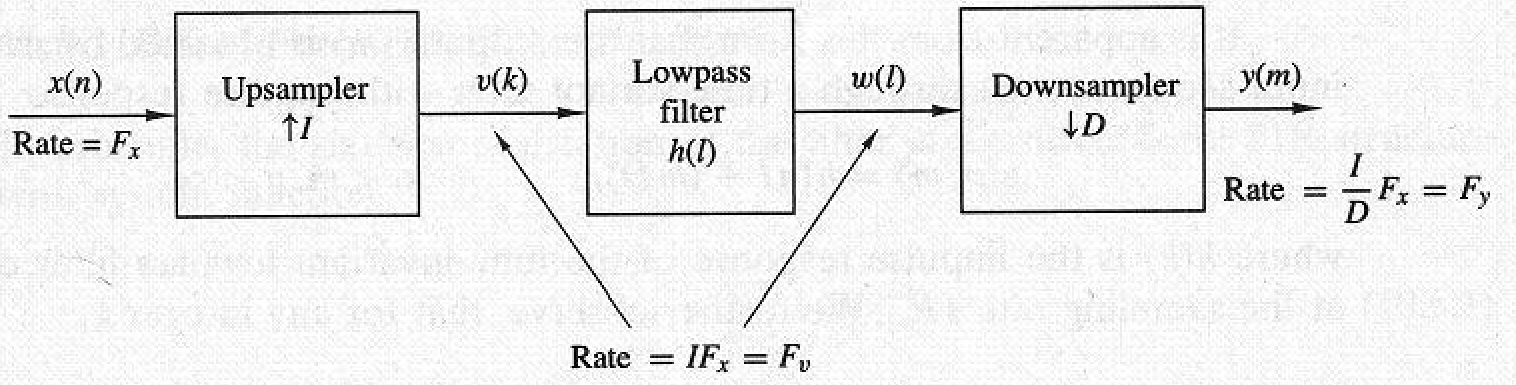


Figure 10.7 Method for sampling rate conversion by a factor I/D.

Proakisand Monolakis_10.7

Multirate DSP – Conversion by I/D

In the time domain, the output of the upsampler is the sequence

$$v(l) = \begin{cases} x(l/I), & l = 0, \pm I, \pm 2I, \ldots \\ 0, & \text{otherwise} \end{cases} \tag{10.4.2}$$

and the output of the linear time-invariant filter is

$$\begin{aligned} w(l) &= \sum_{k=-\infty}^{\infty} h(l-k)v(k) \\ &= \sum_{k=-\infty}^{\infty} h(l-kI)x(k) \end{aligned} \tag{10.4.3}$$

Finally, the output of the sampling rate converter is the sequence $\{y(m)\}$, which is obtained by downsampling the sequence $\{w(l)\}$ by a factor of D. Thus

$$\begin{aligned} y(m) &= w(mD) \\ &= \sum_{k=-\infty}^{\infty} h(mD-kI)x(k) \end{aligned} \tag{10.4.4}$$

Multirate DSP – Conversion by I/D

It is illuminating to express (10.4.4) in a different form by making a change in variable. Let

$$k = \left\lfloor \frac{mD}{I} \right\rfloor - n \tag{10.4.5}$$

where the notation $\lfloor r \rfloor$ denotes the largest integer contained in r. With this change in variable, (10.4.4) becomes

$$y(m) = \sum_{n=-\infty}^{\infty} h\left(mD - \left\lfloor \frac{mD}{I} \right\rfloor I + nI\right) x\left(\left\lfloor \frac{mD}{I} \right\rfloor - n\right) \tag{10.4.6}$$

We note that

$$\begin{aligned} mD - \left\lfloor \frac{mD}{I} \right\rfloor I &= mD \qquad \text{modulo } I \\ &= (mD)_I \end{aligned}$$

Consequently, (10.4.6) can be expressed as

$$y(m) = \sum_{n=-\infty}^{\infty} h(nI + (mD)_I)x\left(\left\lfloor \frac{mD}{I} \right\rfloor - n\right) \tag{10.4.7}$$

Multirate DSP – Conversion by I/D

$$y(m) = \sum_{n=-\infty}^{\infty} h\left(mD - \left\lfloor \frac{mD}{I} \right\rfloor I + nI\right) x\left(\left\lfloor \frac{mD}{I} \right\rfloor - n\right)$$

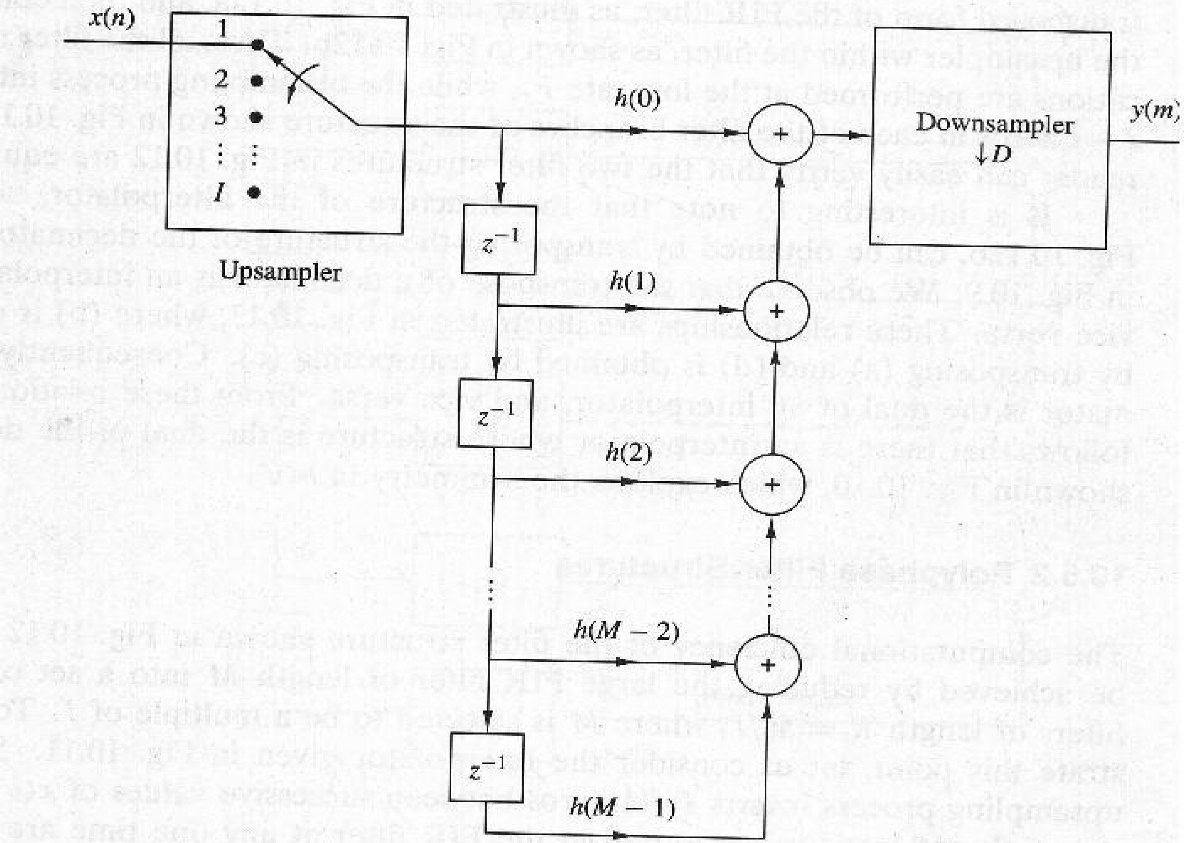
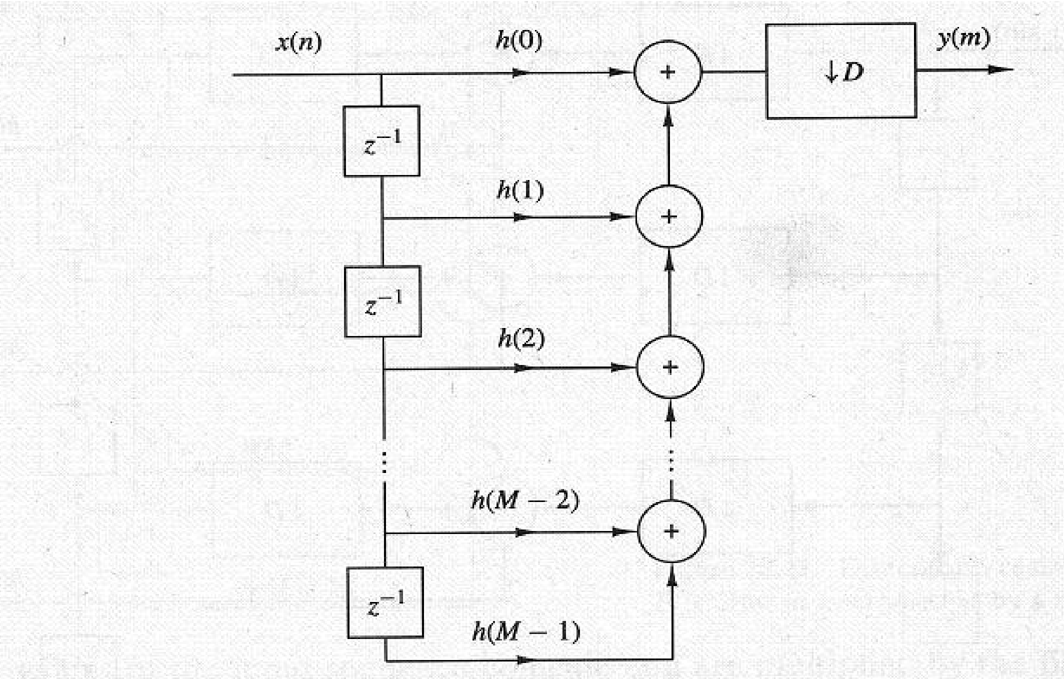


Figure 10.8 Direct-form realization of FIR filter in sampling rate conversion by factor I/D.

Proakisand Monolakis_10.8

Multirate DSP – Decimation by D

Decimation after calculating the output - inefficient



(a)

Proakisand Monolakis_10.9

Multirate DSP – Decimation by D

Efficient Decimation file structure

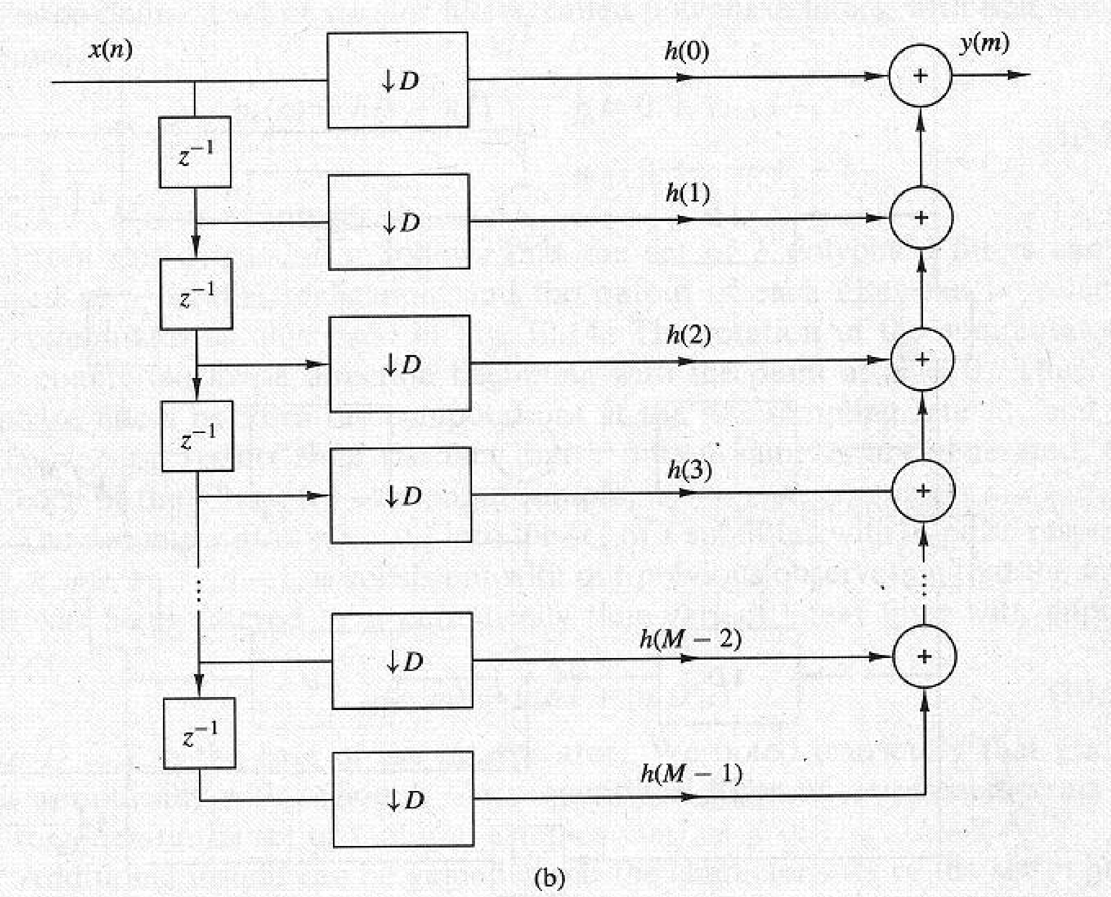
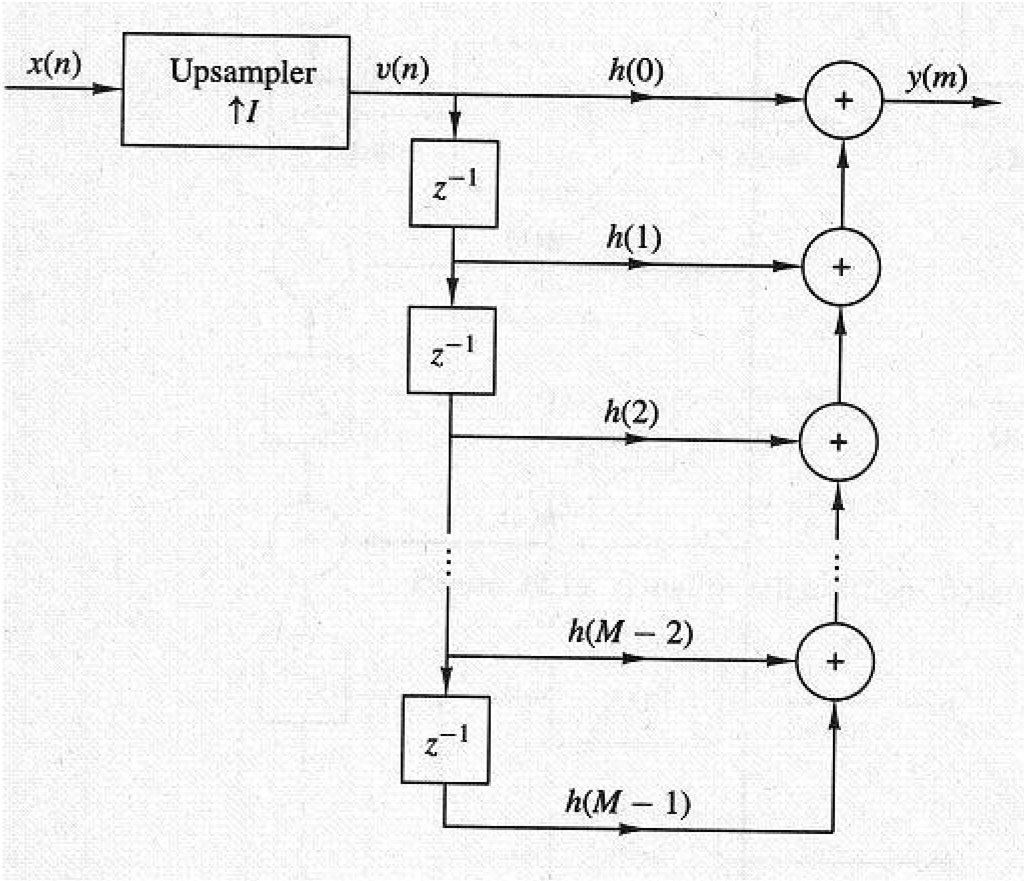


Figure 10.9 Decimation by a factor D.

Proakisand Monolakis_10.9

Multirate DSP – Interpolation by I

Interpolation at input before filter - inefficient



Proakisand
Monolakis_10.11

Multirate DSP – Interpolation by I

Efficient Interpolation within the filter structure

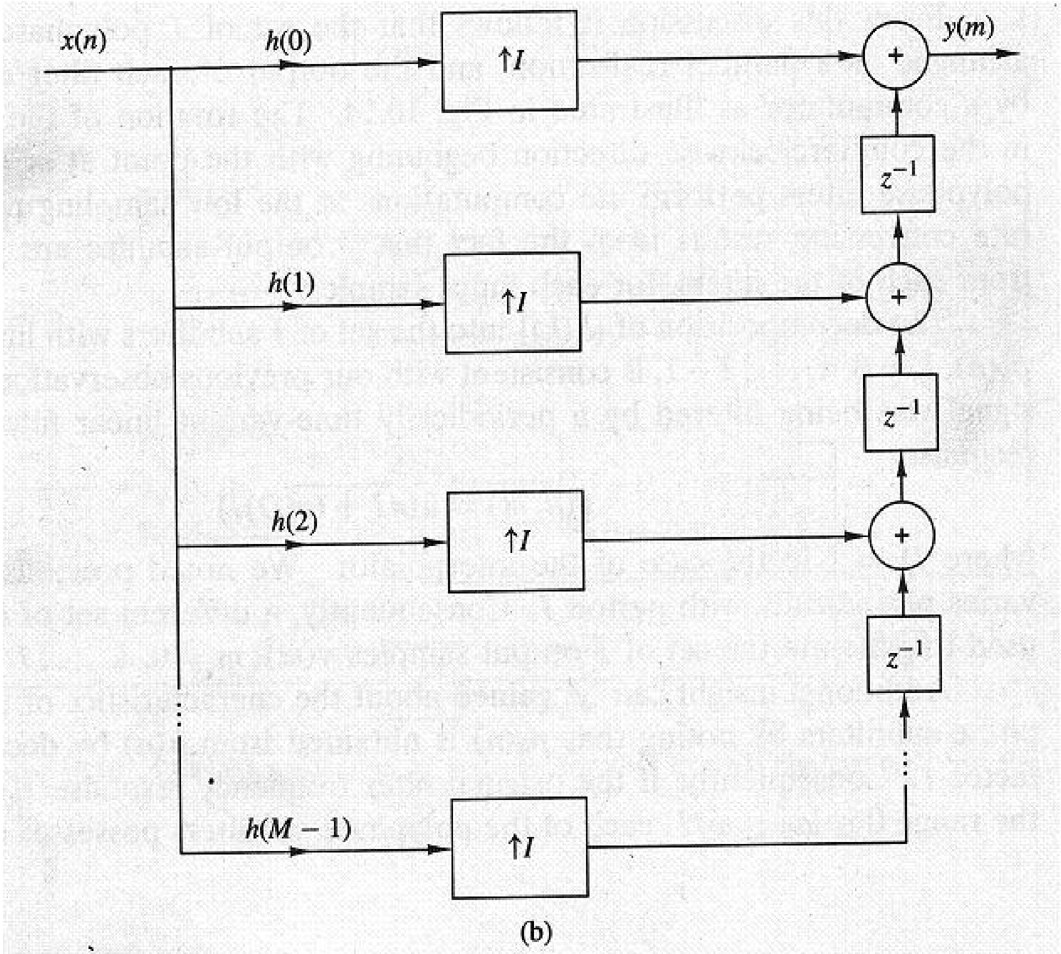


Figure 10.12 Efficient realization of an interpolator.

Proakisand
Monolakis_10.12

Polyphase filter structures - Interpolation

$$p_k(n) = h(k + nI) \qquad k = 0, 1, \ldots, I - 1$$

$$n = 0, 1, \ldots, K - 1$$

where $K = M/I$ is an integer.

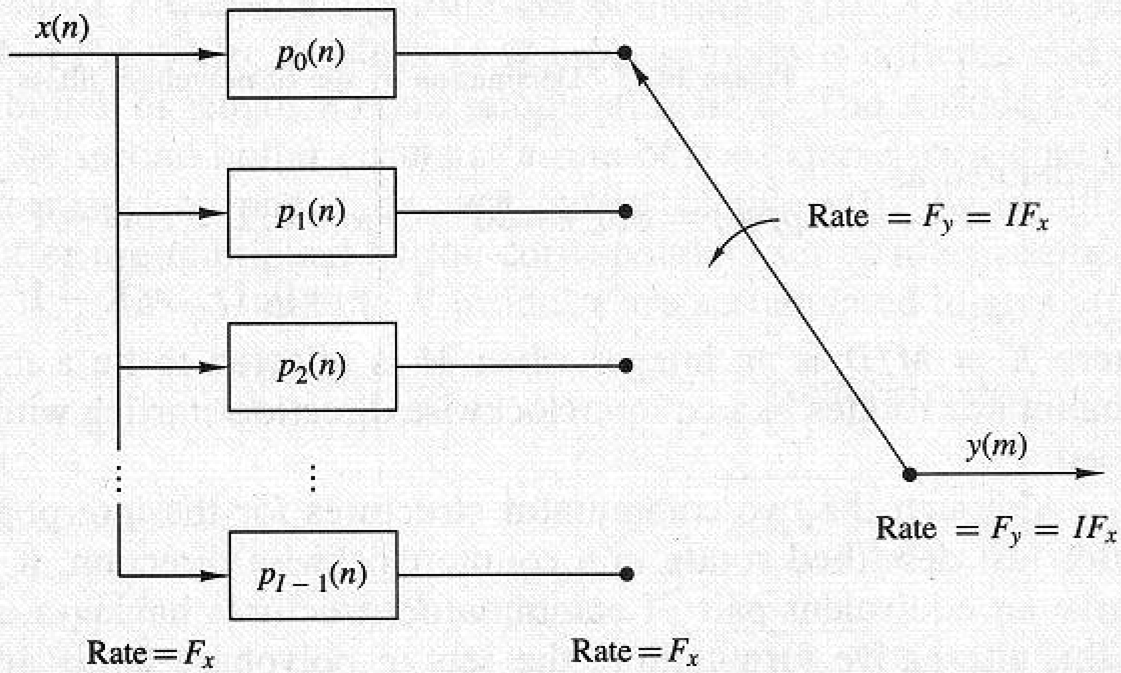


Figure 10.14 Interpolation by use of polyphase filters.

Proakisand
Monolakis_10.14

Polyphase filter structures - Decimation

$$p_k(n) = h(k + nD) \qquad k = 0, 1, \ldots, D-1$$
$$n = 0, 1, \ldots, K-1 \qquad (10.5.4)$$

where $K = M/D$ is an integer when M is selected to be a multiple of D. The commutator rotates in a counterclockwise direction starting with the filter $p_0(n)$ at $m = 0$.

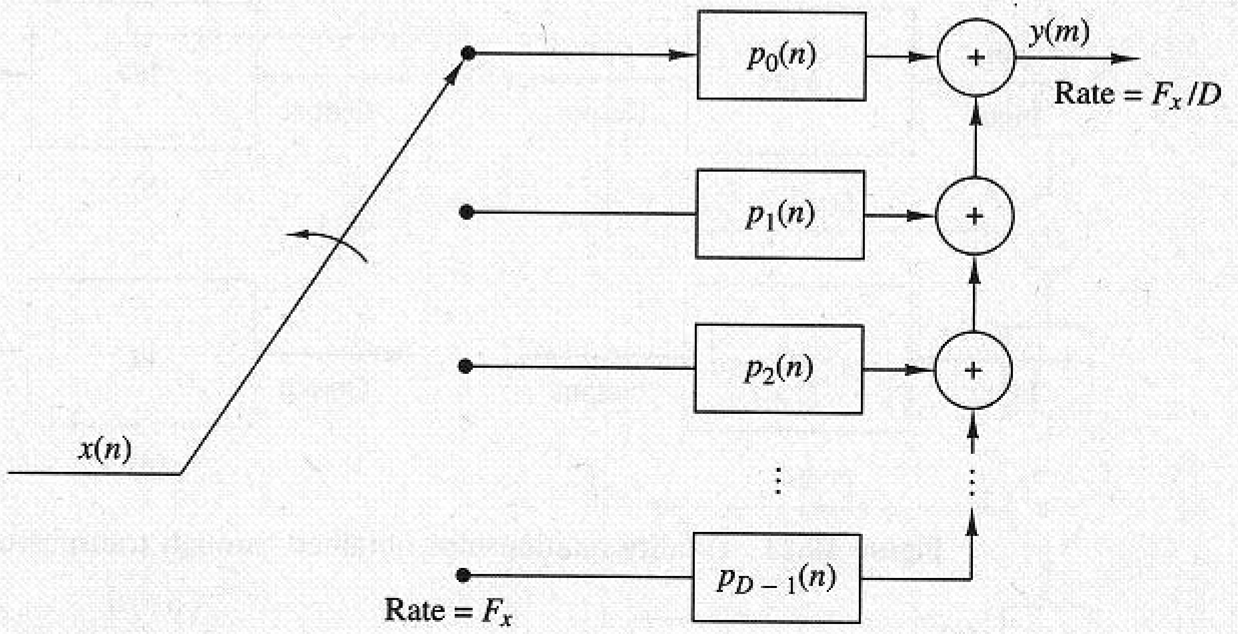


Figure 10.15 Decimation by use of polyphase filters.

Proakisand
Monolakis_10.15

Polyphase filter structures

Although the two commutator structures for the interpolator and the decimator just described rotate in a counterclockwise direction, it is also possible to derive an equivalent pair of commutator structures having a clockwise rotation. In this alternative formulation, the sets of polyphase filters are defined to have impulse responses

$$p_k(n) = h(nI - k) \qquad k = 0, 1, \ldots, I - 1 \tag{10.5.5}$$

$$p_k(n) = h(nD - k) \qquad k = 0, 1, \ldots, D - 1 \tag{10.5.6}$$

for the interpolator and decimator, respectively.

Sampling Rate Conversion by I/D

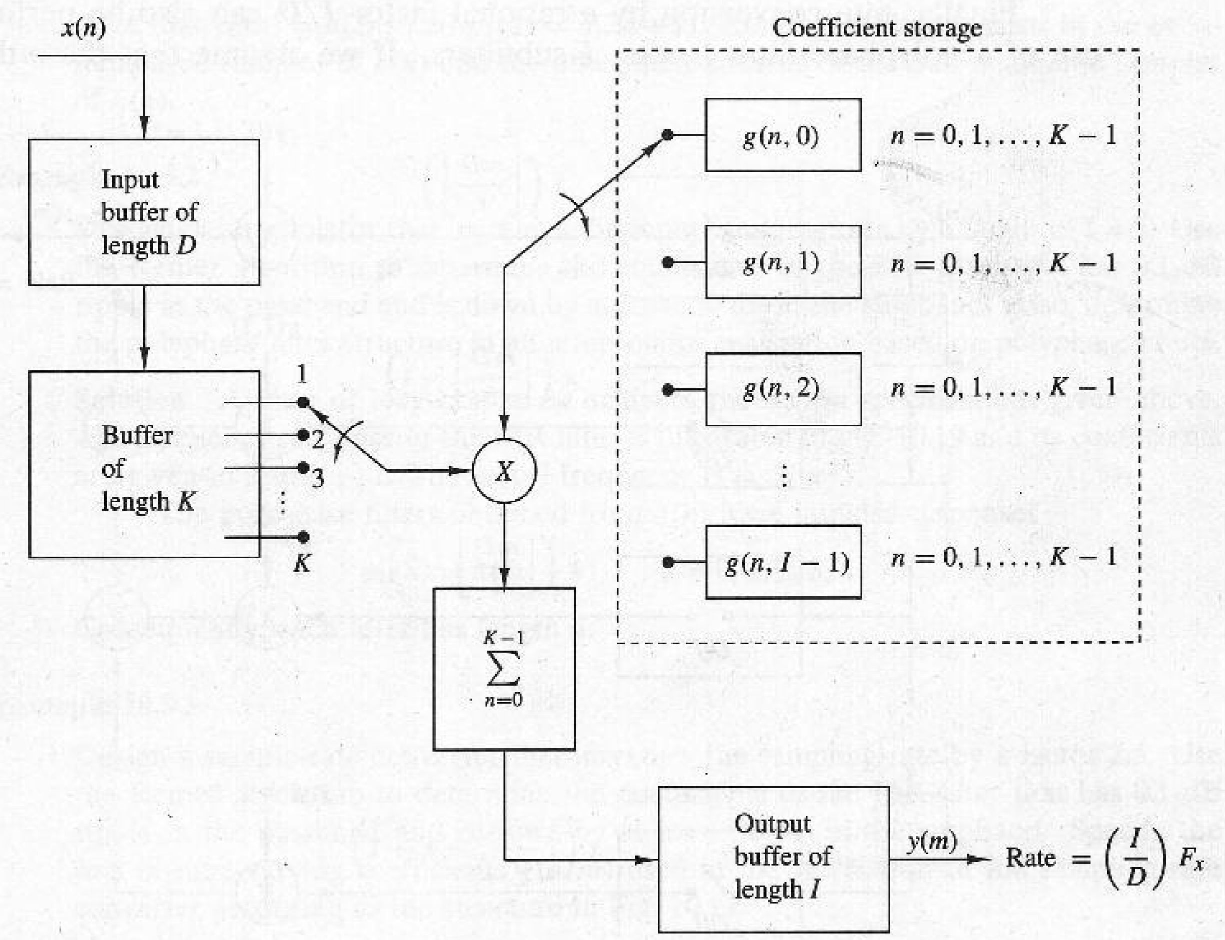


Figure 10.16 Efficient implementation of sampling-rate conversion by block processing.