Corrections and Additions[1] to the book
Topics In Graph Automorphisms & Reconstruction

1. P.2 lines 22: "or, equivalently" becomes "and".

2. P.11 lines 17 & 18: "incident" becomes "adjacent".

3. P.27 Th. 2.7: From the 3rd displayed equation, there are some errors in the exponents. Perhaps the simplest way to correct this without too many alterations is (starting from the first displayed equation for clarity):

$$\begin{aligned}\sum_{\rho\neq \mathrm{id}} P(S_\rho) &= \sum_{g=1}^{t}\sum_{\rho: g(\rho)=g} P(S_\rho)\\ &\leq \sum_{g=1}^{t} n^{2g}\left(\frac{1}{2}\right)^{2g(t-2)/6}\\ &= \sum_{g=1}^{t}\left[n^2 2^{(2-t)/3}\right]^g\\ &= \sum_{g=1}^{t}\left[4^{1/3}n^2 2^{-t/3}\right]^g.\end{aligned}$$

Now $t = n - k > 12(k+1)\lg n$ for sufficiently large n. Therefore $4^{1/3}n^2 2^{-t/6} < \frac{1}{n^{k+1}}$, provided $4^{1/3} < n^{k+1}$. Therefore

and then the rest proceeds as in the book.

4. P.28: Perhaps it would be helpful to add this sentence at the end of the first paragraph: "Because A is symmetric, its eigenvalues are real."

5. P.30 in the definition of a t-arc: "(even consecutive ones)" becomes "(except consecutive ones)".

6. P.35 line -11: "encyclopaediac" becomes "encyclopaedic".

7. P.38 line -4: $(\alpha, \alpha\sigma)$ instead of $(\alpha, g\sigma)$.

[1]With special thanks to Lewis Holland, Alex Scott, Bill Kocay, Virgiglio Pannone, Russell Mizzi and others.

8. Proof of Theorem 3.2, first paragraph. Lines 2 & 3: $\alpha\beta$ instead of $\beta\alpha$. Line 3: missing open bracket at the very beginning. And penultimate line: $\lambda_\alpha(\beta)\sigma$ instead of $\lambda_\alpha(\beta)s$.

9. Theorems 3.3 & 3.4: Missing references:

   R. Frucht. Herstellungen von Graphen mit vorgegebener abstrakten Gruppe. *Compositio Math.*, 6:239–250, 1938.

   I. Z. Bouwer. Section graphs for finite permutation groups. *J. Combin. Theory*, 6:378–386, 1971.

   Another proof for Theorem 3.4 can be found as solution to Problem 12.21 in:

   L. Lovász. *Combinatorial Problems and Exercises, 2nd Ed.* North-Holland, 1993.

10. P.43, penultimate paragraph, last line: v_0 instead of v.

11. P.45 lines 1 and 6: "subgraph" becomes "subgroup".

12. P.57 Fig. 4.1: The orientations of the two 5-cycles should be opposite each other.

13. P.58 line 5: "be" becomes "by"; line 7 "is is" becomes "it is".

14. Proof of Theorem 4.4: First sentence: "orbits" becomes "orbitals". Second sentence "orbit" becomes "orbital".

15. P.73 Exercise 5.1 line 4: "subgroup" becomes "subset".

16. P.73 Exercise 5.2 line 4: α^3 becomes $\alpha\beta^3$.

17. P.78, definition of general products of graphs:

    After the first paragraph insert:
    "Our definition of graph products will be based on the definition of probably the simplest to describe graph product of all, namely the *categorical product*. This is defined as follows. Let G and H be two graphs (directed or undirected). Then the vertex-set of the categorical product $G \times H$ is $V(G) \times V(H)$ and its arc-set is defined by:

    > (a, b) is adjacent to (c, d) if and only if a is adjacent to c and b is adjacent to d."

The next paragraph then becomes:
"Let $\mathcal{G}$ be a class of graphs (directed or undirected). Let I be a set of nonnegative integers. A *twisting graph function* (TGF) is a map $f : \mathcal{G} \to calG$ such that $f(G)$ has the same vertex set as G and in $f(G)$ two vertices are adjacent if and only if the distance between them belongs to I. Note that this definition applies to digraphs as well, where the distance d(a,b) stands for the length of the shortest directed path from a to b."

(Notes: As TGFs are defined in the book, Lemma 6.1 would be false. The above definition fixes this.

Also, for the three examples of TGFs given after the definition, *id, cp, cpl* and *loop*, the set I would be, respectively, $\{1\}$, $\mathbb{N}-\{0,1\}$, $\mathbb{N}-\{0,1\}$ and $\{0\}$.)

Then, in the definition of a general graph product, the small union sign should be a cartesian product, and the subsequent phrase should be: "where all the f_i and f_i' are TGFs and the product means the categorical product of graphs defined above."

18. P.80 proof of Th. 6.4 line 4: Finish the first sentence with "such that x is n an odd cycle."

19. P.81 statement of Lemma 6.6, line 30: end of line should be $d(x,y) = d(x_1,x_2) + d(y_1,y_2)$.

20. P.83 penultimate line of proof of Th. 6.7: $d(a,x) + d(x,b)$ becomes $d(a,x) + d(x,b) \geq d(a,b)$.

21. P.83 Last line of Section 6.3: "regardless of the two factors" should be "regardless of the second factor, provided the first factor is connected".

22. P.83 Statement of Theorem 6.9: G and H should be graphs not digraphs.

23. P.84 lines 1 and 3: (u,v) should be $(u,f(u))$ and $(f(u),f(v))$ should be $(v,f(v))$.

24. P.86 Nešestřil should be Nešetřil. Similarly in Ref. 156.

25. P.87 third para of Sec. 6.7 second sentence: "then" becomes "than".

26. P.88 second para line 3: "consructions" becomes "constructions".

27. P.104 line 2: "reconstrcution" becomes "reconstruction".

28. P.110 Cor. 8.9: The conditions "order at least 7 and minimum degree ar least 3" from Th. 8.8 are required in the statement of the corollary.

29. P.111 last line of the proof of Th. 8.13: "unquely" becomes "uniquely".

30. P.128 Exercise 10.3: $\binom{m-t}{n-3}$ should become $\binom{m-t}{n-t}$. This problem is taken from R. Taylor, "Subgraph identities and reconstruction", *Ars Combin.* **19** (1985), 245–256.

31. P.143 Ref.14: "Automrphism" becomes "Automorphism".

32. Theorem 8.7: As pointed out by Alex Scott, the proof represents the deck as a sum over G_i in S of the number of times G_i occurs as a subgraph of G, whereas this should actually be the number of times it occurs as an *induced* subgraph of G. From $\mathcal{D}(G)$ as obtained in the theorem (all subgraphs on $n-1$ vertices and less than m edges) it is easy to obtain the correct deck, but some more technical work needs to be done, and we give this here. First, let us rename the $\mathcal{D}(G)$ obtained in the theorem as $\mathcal{D}'_0 = \mathcal{D}'_0(G)$ (the use of suffix 0 will be made clear below). We now show how the actual deck, $\mathcal{D}(G)$ is obtained from $\mathcal{D}'_0(G)$:

    Let $\mathcal{D}_1$ be the graph isomorphism types contained in $\mathcal{D}'_0$ (together with multiplicities) such that these graphs are not subgraphs of any other graphs in $\mathcal{D}'_0$. These are clearly induced subgraphs of G and they are in the deck of G; yet they (and hence $\mathcal{D}_1$) need not be the whole deck of G. Represent $\mathcal{D}_1$ as a sum of isomorphism types of graphs with the coefficients representing multiplicities (as in the theorem). Let $\mathcal{D}'_1$ be obtained from $\mathcal{D}_1$ by adding to this sum all the subgraph types obtained by deleting edges in all possible ways and numbers from the graphs in $\mathcal{D}_1$, taking care to determine the correct multiplicities of these subgraphs. Consider the difference between the two sums

    $$\mathcal{D}'_0 - \mathcal{D}'_1.$$

    Determine those graph types contained in this difference such that these are not subgraphs of any other graphs in the difference; these are clearly induced subgraphs of G, and hence in the deck of G. Add these subgraphs (with their multiplicities as they appear in the difference) to $\mathcal{D}_1$ to give $\mathcal{D}_2$.

Repeat the above procedure to give $\mathcal{D}_2'$, etc, until $\mathcal{D}_r' = \mathcal{D}_0'$. When that happens, $\mathcal{D}_r$ is the deck of G, $\mathcal{D}(G)$. □

Perhaps an example will make this last part of the theorem more clear. Let G be a square with a diagonal edge. Then

$$\mathcal{D}_0' = 2K_3 + 8P_2 + 10(P_1 \cup N_1) + 4N_3$$

where K_k is the complete graph on k vertices, P_k is the path on k edges and N_k is the null graph on k vertices. Then,

$$\mathcal{D}_1 = 2K_3$$

$$\mathcal{D}_1' = 2K_3 + 6P_2 + 6(P_1 \cup N_1) + 2N_3$$
$$\mathcal{D}_2 = 2K_3 + 2P_2$$
$$\mathcal{D}_2' = 2K_3 + 8P_2 + 10(P_1 \cup N_1) + 4N_3.$$

And since $\mathcal{D}_2' = \mathcal{D}_0'$, $\mathcal{D}_2$ is the deck of G, which can easily be verified to be correct.

33. Perhaps more specific references for Theorems 10.6 & 10.7 would be, respectively:

    H. Sachs. Bezeihungen zwischen den in einem Graphen enthaltenen Kreisen und seinem charakteristeschen Polynom. *Publ. Math. Debrecen*, 11:119–134, 1964.

    H. Whitney. A logical expansion in mathematics. *Bull. Amer. Math. Soc.*, 38:572–579, 1932.

34. P.135 statement of Theorem 11.2: The summation should run over all $Y \subseteq X$; same thing for P.136, lines 5 and 6.

Last updated 25 April 2005