Tutorial 5 Convolutional Codes

1. (a) Sketch the diagram of a (3,2,1) convolutional encoder with

$$G_o^{(0)}(D) = 1+D;\ G_1^{(0)}(D) = D$$
$$G_o^{(1)}(D) = D;\ G_1^{(1)}(D) = 1$$
$$G_o^{(2)}(D) = 1+D;\ G_1^{(2)}(D) = 1$$

   (b) Alter the encoder such that

$$G_o^{(0)}(D) = 1+D;\ G_1^{(0)}(D) = D$$
$$G_o^{(1)}(D) = 1+D;\ G_1^{(1)}(D) = D$$
$$G_o^{(2)}(D) = 1+D;\ G_1^{(2)}(D) = D$$

2. (a) For the binary convolutional codes below, give the structure of the code (n,k,m).

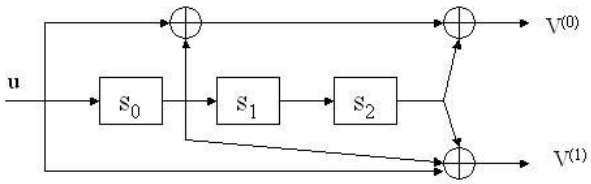

Figure 1

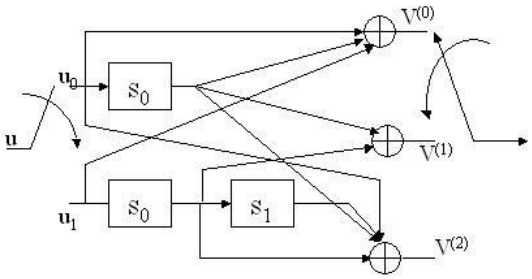

Figure 2

   (b) Hence for each code work out the the generators $G_i^{(j)}(D)$ in terms of the 'delay' element D.
   (c) In each case write down the generators as vectors using octal representation.
   (d) Hence draw the state diagrams for each code.
   (e) What is the constraint length of the codes?

3. (a) What defines a catastrophic convolutional code?
   (b) Derive the generators of Figure 3 below.
   (c) Hence work out the state table.
   (d) Is this a catastrophic code? Give reasons.

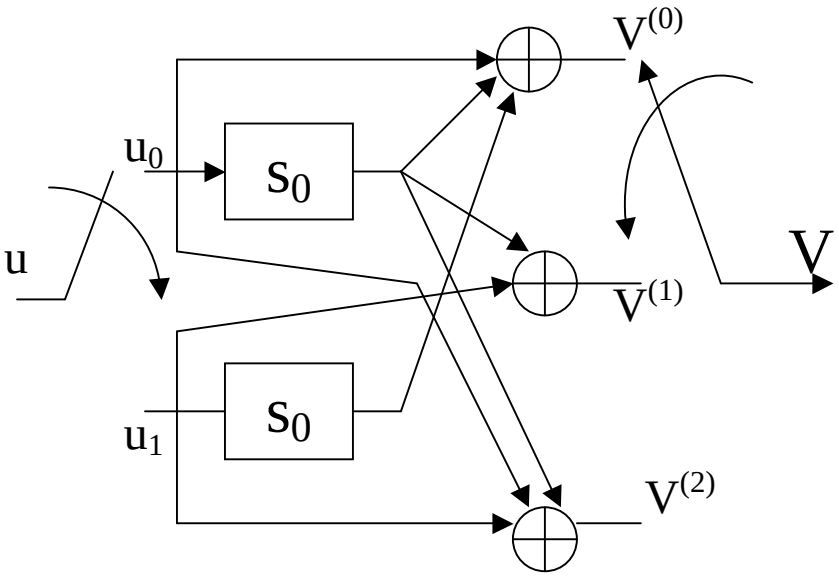

Figure 3

4. For the rate ½ memory-2 convolutional encoder with generators (5,7)
   (i) Obtain the state table and state diagram.
   (ii) Sketch the initial trellis diagram.
   (iii) For an input sequence, 1 1 0 0 1 0 0, work out the source code.
   (iv) Assume that the received bits are 1 1 0 1 1 0 1 1 1 1 0 1 0 0 , use a Trellis diagram and Viterbi decoding, to obtain the code bits transmitted.
   (v) In this case up to which of the bits should the Viterbi decoder be giving a proper answer. Why?

5. What is a punctured convolutional code? How does puncturing affect the free distance of the code?

6. Using the zero-tail construction and the binary memory 2 rate ½ code with generators (5,7), construct a binary linear block code C of dimension k =5. Determine
   (i) the length and minimum Hamming distance of code C
   (ii) the WDS A(x) of C
   (iii) the error performance of C with binary transmission over a BSC channel.