A survey of some open questions in reconstruction numbers

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To the memory of Frank Harary

Abstract

Frank Harary contributed numerous questions to a variety of topics in graph theory. One of his favourite topics was the Reconstruction Problem which, in its first issue in 1977, the Journal of Graph Theory described as the major unsolved problem in the field. Together with Plantholt, Frank Harary initiated the study of reconstruction numbers of graphs. We shall here present a survey of some of the work done on reconstruction numbers, focusing mainly on the questions which this work leaves open.

1 Introduction

All graphs G are assumed to be finite and simple. Graph theoretic terms which are not defined here can be found in [10] except that we here use the terms "vertex" and "edge" for "point" and "line", respectively.

The *deck* of G, denoted by $\mathcal{D}(G)$ is the multiset of vertex-deleted subgraphs $G - v$ for all vertices v of G (we say "multiset" because isomorphic vertex-deleted subgraphs, if there are any, appear in the deck as many times as their multiplicities in G); a vertex-deleted subgraph of G is called a *card* of G. A graph G is *reconstructible from a subdeck S* of its deck if every graph H whose deck contains S is isomorphic to G; when S is the whole deck G is said to be *reconstructible* from the deck. The famous Reconstruction Conjecture (for a recent survey see [15], for example) states that all graphs with at least three vertices are reconstructible from their decks.

A graph G has *ally reconstruction number* $\mathrm{rn}(G) = p$ if p is the size of a smallest (counting multiplicities) subdeck of G from which G is reconstructible. (The ally reconstruction number has usually been called simply the "reconstruction number" in previous work.)

Suppose, moreover, that we are given, apart from a subdeck of $\mathcal{D}(G)$, the information that G is in some class $\mathcal{C}$. Then the *class ally reconstruction number* $\mathcal{C}\mathrm{rn}(G) = p$ of G is the size of a smallest subdeck of G from which, together with the information that G is in $\mathcal{C}$, G is reconstructible; that is, any graph H in $\mathcal{C}$ which contains the subdeck is isomorphic to G.

The *edge-deck* $\mathcal{ED}(G)$ of the graph G is analogously defined as the multiset of edge-deleted subgraphs (*edge-cards*) $G-e$ of G for all edges e of G. Edge-reconstruction is also analogously defined. The Edge-Reconstruction Conjecture states that all graphs with at least four edges are edge-reconstructible.[1]

The *ally edge-reconstruction number* $\mathrm{ern}(G)$ and the *class ally edge-reconstruction number* $\mathcal{C}\mathrm{ern}(G)$ of G are defined in a manner analogous to the ally reconstruction numbers.

Interest in a problem such as the Reconstruction Problem in graph theory could wane because of the sheer difficulty of obtaining significant new results. Alternatively, interest might shift to variants of the problem which open up new avenues. One such variant, which is currently attracting the attention of a number of researchers, is the reconstruction of combinatorial structures other than graphs [1, 25, 26, 27].

In 1985, Frank Harary and Mike Planholt wrote a short paper [13] which introduced such a variant, that is, the notion of reconstruction numbers of graphs. This idea gave a new lease of life to the reconstruction problem, enabling researchers to re-visit graphs which were otherwise known to be reconstructible (some almost trivially) but with new restrictions which brought to light new problems where before there were apparently none. This work also brought up new questions which have remained unanswered. Some of the results obtained also seemed to give more evidence in favour of the truth of the reconstruction conjectures, an important motivation which could encourage researchers to study these graph parameters.

The reconstruction numbers defined above can be seen as a game between two players A and B [22]. Player B is given a graph G and is required to chose, from G's deck, the smallest number of cards to give to A which

[1] The terms "deck" and "card" and the edge-version of the reconstruction problem were introduced by Harary.

suffice for A to determine G—this number is the ally reconstruction number of G.

But players A and B can be adversaries. In this case, B's task would be to find the largest number of subgraphs in the deck of G such that, given to A, they would not be sufficient for A to determine G without ambiguity. Adding 1 to this number gives the parameter that is called the *adversary reconstruction number of* G, denoted by Adv-rn(G), which is the smallest value of k for which no k-subdeck of the deck of G is in the deck of any other graph which is not isomorphic to G—the adversary edge-reconstruction number is analogously defined. It is clear that the ally reconstruction number of a graph is always at most as large as its adversary reconstruction number. In this paper we shall be discussing both the ally and the adversary reconstruction numbers.

2 The ally reconstruction number

Harary and Plantholt, in their paper introducing reconstruction numbers, put forward a number of questions. Amongst these was the conjecture that almost every graph has ally reconstruction number equal to 3. (It is clear that no graph can have ally reconstruction number less than 3.) This result—a stronger version, in fact—was proved, implicitly or explicitly, by a number of authors [21, 22, 5]. We say that almost every graph has a certain property if the proportion of labelled graphs on n vertices which have the property tends to one as n tends to ∞.

Theorem 2.1 *Almost every graph has the property that any three vertex-deleted subgraphs from its deck determine it uniquely. Therefore almost every graph has ally reconstruction number equal to 3.*

An analogous result for edges can be found in Exercise 9.7 of [16].

Theorem 2.2 *Almost every graph has the property that any two edge-deleted subgraphs from its edge-deck determine it uniquely. Therefore almost every graph has ally edge-reconstruction number equal to 2.*

The data in Table 1, obtained by McMullen and Radziszowski [17], gives a very good idea of how strong Theorem 2.1 is.

Theorem 2.1 (Theorem 2.2) follows from the fact that almost all graphs have the property that deleting any three different vertices (two different edges) will give non-isomorphic subgraphs; that is, almost every graph is highly asymmetric. In the next section, after we consider the ally reconstruction number of regular graphs, we shall comment on what light these results could possibly shed on the relationship between graph symmetries and the difficulty of graph reconstruction.

Ally rec no	Order							
	3	4	5	6	7	8	9	10
3	4	8	34	150	1044	12,334	274,666	12,005,156
4		3		4		8		6
5				2		2	2	4
6						2		
7								2

Table 1: *Number of graphs with given order and given ally reconstruction number*

2.1 Regular graphs

The above results not only provide evidence in favour of the Reconstruction Conjectures but are also an invitation to revisit classes of graphs which are known to be reconstructible to try and determine how few of the cards in their decks are really required to determine them. The first such class of graphs which comes to mind is the class of regular graphs; an r-*regular* graph is a graph all of whose vertices have degree r. The reconstructibility of such graphs is a very simple exercise but the determination of their ally reconstruction numbers proves to be a remarkably difficult task.

Myvold [22] has shown that r-regular graphs have ally reconstruction number at most $r+3$ and that this bound is attained by the graph pK_{r+1} consisting of p disjoint copies of K_{r+1}. Asciak [2] further showed that pK_{r+1} is the only r-regular graph with ally reconstruction number $r+3$. This leaves wide open the investigation of the gap between ally reconstruction numbers 3 and $r+3$ for such graphs.

Problem 2.1 *For any integer k between 3 and $r+3$, do there exist r-regular graphs with ally reconstruction number k ?*

Suppose that G is r-regular but not a union of copies of K_{r+1}. What is $f(r)$, the largest possible value of $rn(G)$? (Asciak's result shows that $f(r) \leq r+2$.) Is $\lim_{r\to\infty} r+3/f(r) = \infty$? Is the problem made any easier by letting G be, say, vertex-transitive, or asymmetric?

We note that the function f cannot be a constant independent of r since examples constructed in [17] show that, for r even, $f(r) > r/2+1$.

What light could these problems shed on the nature of the Reconstruction Problem? As pointed out in [22], graphs which are asymmetric should be easier to reconstruct, yet symmetric graphs (even those which are at least regular), which should present a stiffer challenge, are simple to reconstruct. However, the notion of ally reconstruction numbers seems to put this issue into better perspective. The ally reconstruction number of regular graphs can be very high and determining it in general is difficult. On the other hand, Theorem 2.1 holds because most graphs are very asymmetric (removing different sets of three vertices gives non-isomorphic subgraphs) and such asymmetric graphs must have ally reconstruction number equal to 3.

The question of the ally edge-reconstruction number of regular graphs does not seem to be easier. In [2] it was shown that for an r-regular graph G, $\mathrm{ern}(G) \leq r+2$, but no regular graphs attaining this bound have, as yet, been found. In fact, the following conjecture is made in [2]. (Note that a cycle has edge-reconstruction number equal to 3.)

Conjecture 2.1 *Let G be an r-regular graph, $r \geq 3$. Then $ern(G) \leq 2$.*

2.2 Disconnected graphs

The next class of graphs which is not difficult to reconstruct is that of disconnected graphs. Myrvold [23] and Molina [19] have shown that the ally reconstruction number of a disconnected graph is 3 unless all components are isomorphic—in the latter case it could be $c+2$ where c is the order of every component. Myrvold also showed that this upper bound can be attained when the graph is pK_c, that is, p copies of the complete graph K_c.

Here too, Asciak and Lauri [4] have shown that Myrvold's example is the only one that attains the upper bound $c+2$ and that, moreover, there are no disconnected graphs with ally reconstruction number $c+1$ because if $\mathrm{rn}(G) \geq c+1$ (for G disconnected with all components isomorphic), then G must be pK_c and therefore has ally reconstruction number $c+2$.

Here again, the gap between 3 and c, the possible values of the ally reconstruction number, cries out to be investigated.

Problem 2.2 *For any integer k between 3 and c, do there exist disconnected graphs with all components isomorphic and order c with ally reconstruction number k?*

Suppose that G is disconnected with all components isomorphic and of order c and suppose that it is not equal to a union of copies of K_c. What is $g(c)$, the largest possible value of $rn(G)$? (Asciak's and Lauri's result shows that $g(c) \leq c$.) Is $\lim_{c\to\infty} c/g(c) = \infty$? Is $g(c)$ a constant independent of c?

A somewhat analogous situation holds for the ally edge-reconstruction number of disconnected graphs with at least two non-trivial components. Molina [20] has shown that if the graph does not have all components isomorphic nor all components isomorphic to K_3 or $K_{1,3}$ then its edge-reconstruction number is at most 3, whereas if all components are isomorphic then the edge-reconstruction number can be as large as $e + 2$ where e is the number of edges in each component. Using the previous results on the ally reconstruction number of disconnected graphs and the use of line-graphs, Asciak [2] simplified some of the proofs of Molina and also showed that if such a disconnected graph G has $\mathrm{ern}(G) \geq e + 1$ then every component of G is $K_{1,e}$. Therefore similar questions arise here as for disconnected graphs. However, the situation here seems to be more tractable and [3] describes in more detail what disconnected graphs have ern equal to 2 or to 3, and also reports on the following conjecture.

Conjecture 2.2 *If G is a disconnected graph with $ern(G) > 3$ then all components are isomorphic and edge-transitive.*

2.3 Trees

The first non-trivial graphs which were shown to be reconstructible were trees. Myrvold [24] showed that the ally reconstruction number of a tree is 3. Actually, Harary and Lauri [12] had earlier proved the weaker result that if $\mathcal{C}$ is the class of trees then, for any tree T, the class ally reconstruction number $\mathcal{C}\mathrm{rn}(T)$ is at most 3—but in their paper, in most junctures of the proof, $\mathcal{C}\mathrm{rn}(T)$ was shown to be 2, and they made the following conjecture.

Conjecture 2.3 *The class ally reconstruction number of a tree is at most 2.*

Harary and Lauri also considered briefly the class ally edge-reconstruction numbers of trees and found, in spite of the above conjecture, that the trees in Figure 1 have class ally edge-reconstruction number equal to 3.

So the question now is the following.

Problem 2.3 *Are the trees in Figure 1 the only ones with class ally edge-reconstruction number greater than 2?*

Molina [18] has shown that the ally edge-reconstruction number of a tree is at most 3. It is not clear, however, which trees have ally edge-reconstruction number equal to 2.

Problem 2.4 *Characterize trees which have ally edge-reconstruction number equal to 2.*

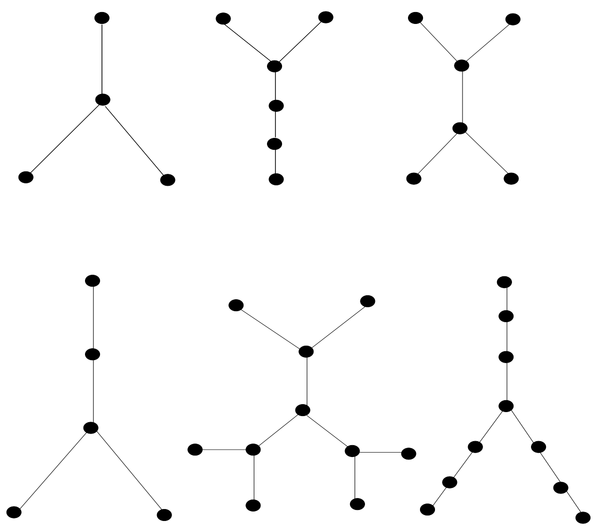

Figure 1: *Are these the only trees with class ally edge-reconstruction number 3?*

2.4 Maximal planar graphs

Maximal planar graphs were shown to be reconstructible by Fiorini and Lauri [8, 14]. Then Harary and Lauri [11] showed that if $\mathcal{C}$ is the class of maximal planar graphs then, for any maximal planar graph G, $\mathcal{C}\mathrm{rn}(G) \leq 2$.

One question here is the following.

Problem 2.5 *What is the class edge-reconstruction number of a maximal planar graph?*

The significance of this and similar problems within the wider context of the Reconstruction Problem will be discussed below.

3 The adversary reconstruction number

The following result follows immediately from the first parts of Theorems 2.1 and 2.2 respectively.

Theorem 3.1 *Almost every graph has adversary reconstruction number equal to 3 and almost every graph has adversary edge-reconstruction number equal to 2.*

For some time, most of the work done on adversary reconstruction numbers could be found in [22, 9]. But recently, significant advances have been reported in [6].

Since the adversary reconstruction number of a graph G is simply one more than the number of cards that G can have in common with any other non-isomorphic graph H, we shall usually address this problem in terms of the number of cards in common between two graphs; this can, in a sense, be viewed as the extent to which two non-isomorphic graphs are similar to each other.

3.1 Tree / unicyclic graph pair

In order to tackle this difficult problem Myrvold concentrated on the adversary reconstruction number of trees, particularly the largest possible number of cards in common between a tree and a non-tree. It is clear that there can be at most two cards in common between a tree and a graph with more than one cycle. Therefore in order to obtain the maximum number of cards in common between a tree and a non-tree, one must consider trees and unicyclic graphs. The following solution to this problem is given in [6].

Theorem 3.2 *A tree and a unicyclic graph on n vertices ($n \geq 19$) can have at most*

$$\lfloor \frac{2}{5}(n+1) \rfloor$$

cards in common.

A family of graphs attaining this bound can be constructed as follows [6]. For $n = 5p+4$, let G be the tree obtained from the path $v_1, v_2, \ldots, v_{3p+2}$ by adding a pair of endvertices to each of v_{3j+2} for $0 \leq j \leq p$, and let H be the graph obtained from the cycle $w_0, w_1, \ldots, w_{3p+2}, w_0$ by adding a pair of endvertices to each of w_{3j+2} for $0 \leq j \leq p-1$, and a single endvertex to w_{3p+2}. For $0 \leq j \leq p$ the removal of one of the endvertices adjacent to v_{3j+2} gives a card isomorphic to $H - w_{3j+1}$ and $H - w_{3p-3j}$. So the number of cards in common between G and H is $2(p+1) = \frac{2}{5}(n+1)$.

The following theorem, a proof of which can be found in [9], was first established by Myrvold and is also reported in [6].

Theorem 3.3 *A connected graph and a disconnected graph on n vertices can have at most $\lfloor \frac{n}{2} \rfloor + 1$ cards in common.*

Therefore from Theorems 3.2 and 3.3 the following result follows.

Theorem 3.4 *For $n \geq 19$, whether a graph of order n is a tree can be determined from any $\lfloor \frac{n}{2} \rfloor + 1$ of its cards.*

Francalanza [9] also considered the number of edge-cards in common between a tree and a unicyclic graph plus an isolated vertex. She proved the following.

Theorem 3.5 *A tree and a unicyclic graph with an isolated vertex, both on n vertices, can have at most $\frac{n}{2}+1$ edge-cards in common.*

Bowler, Brown and Fenner [7] observe that modifying the previous example gives a tree and a unicyclic graph plus an isolated vertex, both on n vertices, with $n/2$ edge-cards in common. They make the following conjecture.

Conjecture 3.1 *The maximum number of edge-cards in common between a tree and a unicyclic graph with an isolated vertex, both on n vertices, is at most $\frac{n}{2}$.*

Note that this conjectured upper bound on the number of edge-cards in common between a tree and a non-tree on n vertices is larger than the upper bound of Theorem 3.2 on the number of cards in common between a tree and a non-tree on n vertices.

It should also be observed here that the trees and unicyclic graphs which attain the bounds given in this section the have a very particular structure. The trees are *caterpillars*, that is, trees the deletion of whose endvertices gives a path, and the unicyclic graphs are what Myrvold and Francalanza call *sunshine graphs*, that is, unicyclic graphs the deletion of whose endvertices gives a cycle.

3.2 Tree pairs

All these partial results and conjectures certainly highlight the main problem here.

Problem 3.1 *How many cards in common can two non-isomorphic trees on n vertices have? Can the adversary edge-reconstruction number of a tree be larger than its adversary reconstruction number?*

The origin of this problem goes back to the Problem Seminar of the 16th Southeastern International Conference on Combinatorics, Graph Theory, and Computing, in Boca Raton in 1985, when Alan Schwenk conjectured that two trees on n vertices can have at most $\lfloor n/2 \rfloor$ cards in common. This conjecture is, however, not true as the next example from [6] shows.

Let

$$\begin{aligned} G^* &= K_{1,p-1} \cup K_{1,p+1} \cup K_{1,p+1} \\ H^* &= K_{1,p} \cup K_{1,p} \cup K_{1,p+1}. \end{aligned}$$

Let G be the tree obtained from G^* by adding a new central vertex and three new edges joining the new vertex to the three cutvertices of G^*. Similarly, construct H from H^*. These two trees on $n = 3p + 5$ vertices have $2p = \frac{2}{3}(n-5)$ cards in common.

This family of tree pairs has the highest known number of cards in common between non-isomorphic trees. A similar construction in [6] gives examples of pairs of trees on n vertices with the same degree sequence and $\frac{2}{3}(n+1-2\sqrt{3n-6})$ cards in common.

3.3 General graph pairs

Of course, the most general problem for adversary reconstruction numbers is the following, a solution of which would settle the Reconstruction Conjecture.

Problem 3.2 *How many cards in common can two non-isomorphic graphs on n vertices have?*

The best result known to date regarding this problem is again given found in [6] where the next theorem is presented. First we require a definition. A *2UC graph pair* is a pair of non-isomorphic graphs, G and H, on n vertices, at least one of which is disconnected, such that in G or in H there are at least two components which cannot be matched with the components of the other graph by isomorphism. A particular example is when G is connected and H is disconnected. ("2UC" stands for "Two Unmatched Components".) The motivation behind this definition is that if A and B are two non-isomorphic connected graphs with the same deck (hence counterexamples to the Reconstruction Conjecture) and on $n-1$ vertices, then $G = A \cup K_1$ and $H = A \cup K_1$ have $n-1$ cards in common.

Theorem 3.6 *Two 2UC graphs can have at most*

$$2\lfloor\frac{1}{3}(n-1\rfloor$$

cards in common.

For $n \geq 22$ and $n \equiv 1(\text{mod}3)$, Bowler, Brown and Fenner give the following infinite family of pairs of 2UC graphs attaining this bound:

$$\begin{aligned} G &= K_{p-1} \cup K_{p+1} \cup K_{p+1} \\ H &= K_p \cup K_p \cup K_{p+1}. \end{aligned}$$

They also show that this pair is unique for the given values of the parameter n. Note that although G and H are disconnected, their complements are connected and also have the same number of cards in common.

More examples are given in [6] including uniqueness of some families of pairs attaining the upper bound in Theorem 3.6. This work also gives an example of pairs of 2UC graphs with $n = 3p^2 - 2, (p \geq 3)$, having the same degree sequence, and

$$\frac{2}{3}(n + 5 - 2\sqrt{3n+6})$$

cards in common; this number is smaller than the upper bound in Theorem 3.6. We therefore single out the following problem.

Problem 3.3 *How many cards in common can two 2UC graphs on n vertices with the same degree sequence have?*

Motivated by Theorem 3.6, Bowler, Brown and Fenner make the following conjecture which, of course, is a considerable strengthening of the Reconstruction Conjecture.

Conjecture 3.2 *For large enough n, the maximum number of cards in common between two non-isomorphic graphs on n vertices is*

$$2\lfloor\frac{1}{3}(n-1)\rfloor.$$

Therefore for sufficiently large n, every graph on n vertices can be reconstructed from any $2\lfloor\frac{1}{3}(n-1)\rfloor + 1$ cards.

Finally, the results in [6] encourage us to pose the following problem.

Problem 3.4 *Investigate the edge-adversary reconstruction number of graphs.*

4 Relation between vertex and edge reconstruction numbers

While the problem of reconstructing from the deck is more difficult than reconstructing from the edge-deck (in fact, if a graph without isolated vertices is reconstructible then it is edge-reconstructible; see [16], for example), the relationship between reconstruction numbers and edge-reconstruction numbers is not that clear. In fact it sometimes happens that more edge-deleted subgraphs are required for unique reconstruction than vertex-deleted subgraphs. For example, a maximal planar graph with minimum degree at least 4 has $\mathcal{C}$rn equal to 1 (see [11]) but no maximal planar graph can have class edge-reconstruction number equal to 1. Also, although Harary and Lauri have conjectured [12] that the class ally reconstruction number of a tree is at most 2, they have found the six trees in Figure 1 which have class ally edge-reconstruction number equal to 3. Even when it comes to the adversary reconstruction number of trees we have seen that it seems that the edge number could be larger than the vertex number.

This suggests the following problem which seems to give a certain interest to the study of reconstruction numbers which is independent of the usual versions of the Reconstruction Problem.

Problem 4.1 *For a class $\mathcal{C}$ of graphs and a graph G in $\mathcal{C}$, what is the relationship between $rn(G)$ and $ern(G)$ and between $\mathcal{C}rn(G)$ and $\mathcal{C}ern(G)$? What is the relationship between adv-$rn(G)$ and the edge-adversary reconstruction number of G?*

5 Conclusion

Reconstruction numbers might be a strong tool for providing evidence to support or reject the Reconstruction Conjecture. Also, the reconstruction number point of view opens up interesting and difficult questions in situations which did not seem to present much difficulty as far as the Reconstruction Problems were concerned. Some of the problems raised, like Problem 4.1, have no counterparts in the Reconstruction Problems. The difficulty of others, like Problem 2.1, seem to give a better insight on the question of how hard it is to reconstruct a particular class of graphs. And we believe that the full solution of some of these problems, like Problem 2.2, might be as difficult as the proving the Reconstruction Conjecture itself.

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