Tutorial 4 BCH Codes

1. A (15,7) BCH code is generated by $g(X) = 1+ x^4+ x^6+ x^7+ x^8$.
   - (i) What is the parity check polynomial h(X), and what are the parameters of the dual cyclic code it generates?
   - (ii) What is the set of roots from $GF(2^4)$ of g(X)?
   - (iii) What is the parity check polynomial h(x), and what are the parameters of the dual cyclic code it generates?
   - (iv) Draw a block diagram of a systematic encoder and syndrome generator for the (15,7) BCH code.

2. A (15,5) binary BCH code has α, α^3, α^5, as roots of the code generator polynomial, where α is a primitive element of $GF(2^4)$.

   - (i) Find the generator polynomial g(X).
   - (ii) Find the parity check polynomial h(X) of this code.
   - (iii) Draw the block diagrams of the k-stage, and (n-k) stage encoders.

3. For the (15,5) BCH code, using g(X) in question 2 above,
   - (i) Determine the codeword v(X) generated by the information message u = 10101.
   - (ii) Suppose the received word has three errors given by the error polynomial $e(X) = X + X^5+ X^{12}$.
   - (iii) Compute the received codeword r(X) = v(X) + e(X), and use r(X) to compute the syndromes S_i , $1 \leq i \leq 6$.
   - (iv) Describe a procedure that can be used to find the location of the errors in r(X), using the syndromes S_i .

4. (a) Distinguish between Reed Solomon codes and BCH codes.
   (b) Distinguish between BCH codes and Cyclic codes.

5. Starting with $p(\alpha) = 1 + \alpha + \alpha^4 = 0$, where α is an element of $GF(2^4)$, and $\alpha^{15}=1$, generate the set of sixteen elements over the multiplicative operation of $GF(2^4)$, in terms of their polynomial representation.
   Hence determine the primitive roots over $GF(2^4)$ for the polynomial $1+X+X^4$, governed by $X^{15}+1=0$.
   Can this set of primitive roots be used to determine the error correction capability of the BCH code with $g(X) = 1+X+X^4$?