Digital Signal Processing CCE 4201

The textbook Digital Signal Processing by J.G. Proakis and D.G. Manolakis is allowed.

Paper duration 2.5 hours

Answer ANY THREE questions

1 (a) Figure 1 shows a 2 pole 2 zero diagram.

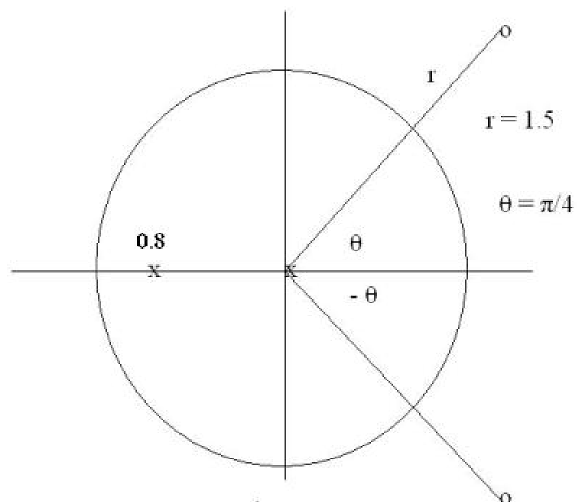

Figure 1

(i) Determine the transfer function in the z-domain. (3 marks)

(ii) Hence sketch a possible implementation of the system and determine the corresponding difference equation. (6 marks)

(iii) Is the system stable? Give reasons. (3 marks)

(iv) The system is a maximum phase system. Sketch the pole-zero diagram of a minimum phase system with the same magnitude response. (3 marks)

(b) Show using suitable block diagrams how the cepstral coefficients of a signal are obtained. (6 marks)

(c) Given a four stage lattice filter with coefficients

$K_1 = \frac{1}{3}$, $K_2 = \frac{1}{4}$, $K_3 = \frac{1}{4}$, and $K_4 = \frac{1}{3}$,

Determine the FIR filter coefficients for the direct form structure. (12 marks)

2 Given the eight-point DFT, X(k) of the sequence

$$x(n) = 2 \qquad 0 \leq x(n) \leq 3$$
$$= 0 \qquad 4 \leq x(n) \leq 7$$

compute the DFT of the sequence y(n), in terms of X(k), given by

$$y(n) = 2 \qquad 0 \leq y(n) \leq 2$$

$$= 0 \qquad 3 \leq y(n) \leq 6$$

$$= 2 \qquad y(n) = 7$$

(7 marks)

(b) The system in (a) above is changed by adding a further eight zeroes, i.e

$$x(n) = 0 \qquad 8 \leq y(n) \leq 15$$

Suggest a relationship, giving reasons, between the DFT of the new sequence, and the DFT of the original sequence.

(9 marks)

(c) An FFT uses in place computation using the decimation-in-frequency algorithm on a 1024 point FFT. Assuming the output 1024 point array is stored from element 0 to element 1023, what is the position in the array of the frequency response for k = 349 ? (4 marks)

(d) Let $x_a(t)$ be an analogue signal with a bandwidth of 4 kHz. An FFT is required to compute the spectrum of the signal with a resolution less than 40 Hz. Determine

(i) the minimum sampling rate; (3 marks)

(ii) the minimum FFT size for the required resolution (5 marks)

(iii) the minimum time length of the analogue signal record. (5 marks)

3 (a) (i) A Butterworth filter has a 3-dB cut off frequency of 4 kHz and an attenuation of 80 dB at a frequency of 4.3 kHz. Calculate the minimum required order for the filter. (8 marks)

(ii) What order would be required if a Type – I Chebyshev filter is used to obtain the specifications in (a) above assuming that a 1-dB ripple is allowed in the passband. (10 marks)

(iii) Give reasons for the difference in the order necessary for the Chebyshev filter compared to the Butterworth filter. (5 marks)

(b) A long speech signal is segmented into blocks of 20 ms. to be analysed. Part of the analysis requires the use of a Hamming window.

(i) What is the Hamming window ? (3 marks)

(ii) Why is a Hamming Window preferred over a rectangular window.

(4 marks)

(iii) Assuming a sampling rate of 8 kHz, what is the size of a suitable Hamming window for the block of 20 ms? (3 marks)

4 (a) Figure 2 shows the Haar wavelet smoothing filter at $\Phi_{0,0}$ and three other instances of a Haar wavelet function at different time and scaling.

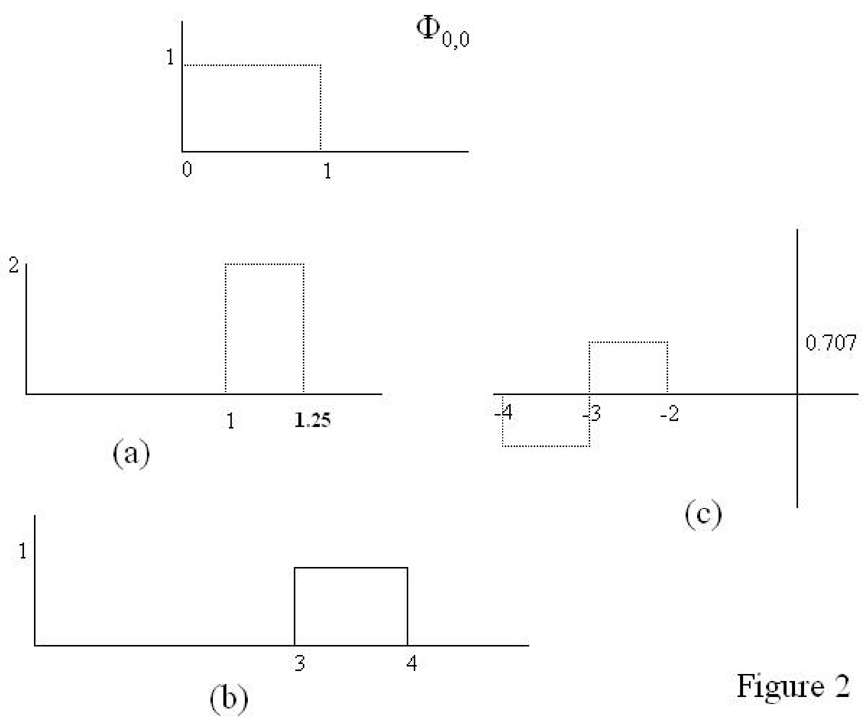

Figure 2

Derive the Haar wavelet function as either $\Phi_{i,j}$ or $\Psi_{i,j}$, as appropriate, for each of the three waveforms (a), (b) , (c), in Figure 2.

(14 marks)

(b)An FIR lowpass filter with 48 coefficients is used to limit the frequency range of a signal to one-sixth of its original value.

(i) Sketch a diagram relating the frequency response of the original waveform to the frequency response after decimation by a factor of 6. (3 marks)

(ii) A polyphase structure is used for the decimation, using the coefficients of the 48 point FIR filter, denoted h(0) to h(47). How many polyphase filters are required? (2 marks)

(iii) For each polyphase filter, give the elements h(i) from the original FIR filter that form part of the particular polyphase filter. Give reasons for your answer. (3 marks)

(iv) Assuming an input sequence x(n) and an output sequence y(m), derive the first four outputs y(m), m = 0 to m=2, in terms of x(n) and h(n), indicating from which polyphase filter each term is being drawn. (6 marks)

(c) Changing the sampling rate of a DSP signal in the digital domain normally uses a rational fraction. Why is a rational fraction preferred?

(5 marks)