



## MFE 3100 Quality Management & Control

### Process Control Using Variable Data

MFE 3100 – Quality Management and Control

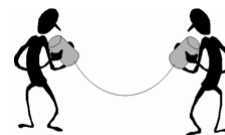
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## A FEW REMINDERS



- Set your Mobile Phones to **Silent**
- Respect your colleagues – no *private* conversations
- Be on time



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## LECTURE OUTLINE



- Introduction to SPC for Variable Data
- Process Control by Variables
- Process Capability
- Case Studies



## INTRODUCTION

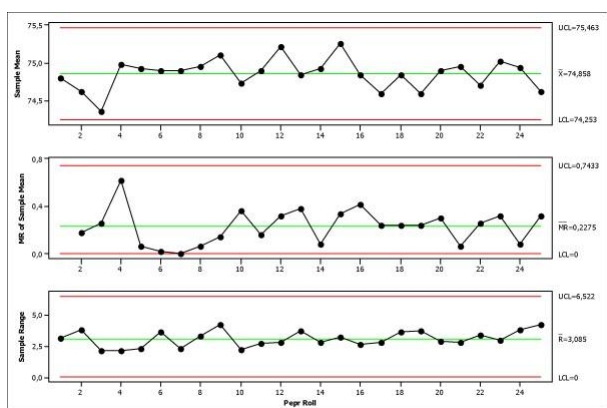




# INTRODUCTION



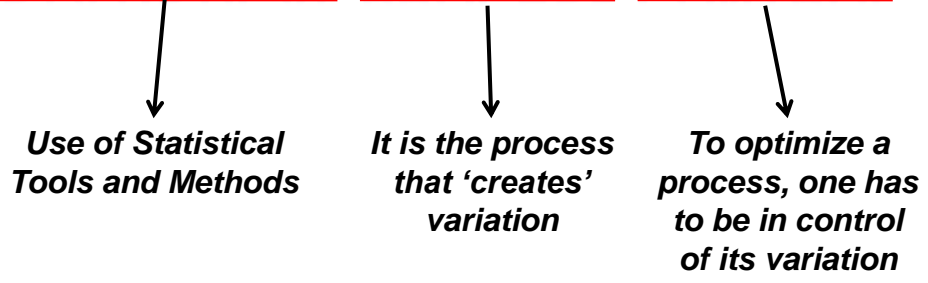
## What is SPC?



# WHAT IS SPC?



## STATISTICAL PROCESS CONTROL





## A BRIEF HISTORY OF SPC



- Shewhart believed that there were 2 variations in a process:
  - Variations which were 'Natural' and inherent to the process.  
**(Common Causes of Variation)**
  - Uncontrolled variation that is not present in the process causal system all the time  
**(Special Causes of Variation)**



SPC



## STATISTICAL PROCESS CONTROL

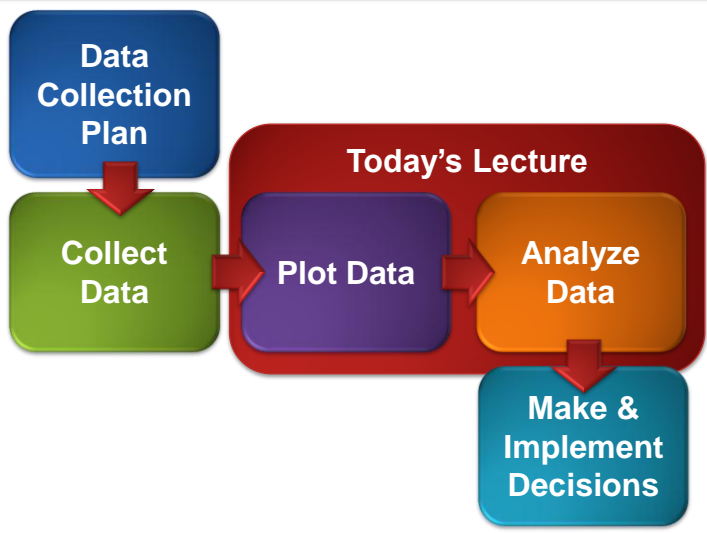


# STATISTICAL PROCESS CONTROL HOW?

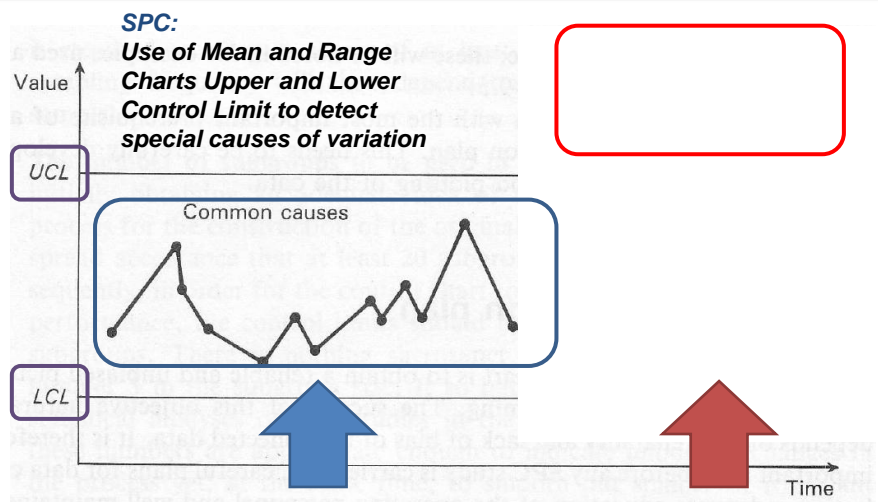




# STATISTICAL PROCESS CONTROL



# SPC CONTROL CHARTS





# PROCESS CONTROL BY VARIABLES

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## STEPS IN ASSESSING PROCESS STABILITY



### 1. Collect Data



## STEEL ROD LENGTH EXAMPLE

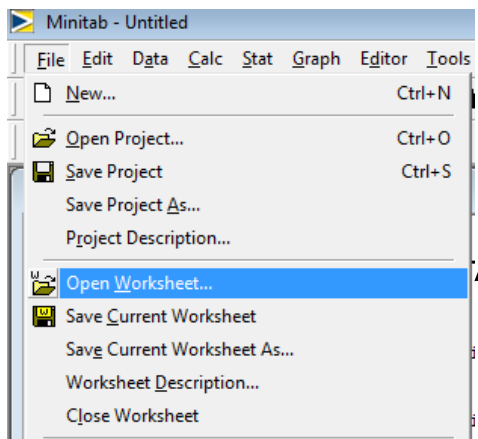
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MINITAB



Open Worksheet that contains data



SAMPLE DATA – STEEL ROD LENGTH EXAMPLE



|         | C1       | C2       | C3       | C4  |
|---------|----------|----------|----------|-----|
| Sample1 | Sample 2 | Sample 3 | Sample 4 |     |
| 1       | 144      | 146      | 154      | 146 |
| 2       | 151      | 150      | 134      | 153 |
| 3       | 145      | 139      | 143      | 152 |
| 4       | 154      | 146      | 152      | 148 |
| 5       | 157      | 153      | 155      | 157 |
| 6       | 157      | 150      | 145      | 147 |
| 7       | 149      | 144      | 137      | 155 |
| 8       | 141      | 147      | 149      | 155 |
| 9       | 156      | 150      | 149      | 156 |
| 10      | 145      | 148      | 152      | 154 |
| 11      | 151      | 150      | 154      | 153 |
| 12      | 155      | 145      | 152      | 148 |
| 13      | 152      | 146      | 152      | 142 |
| 14      | 144      | 160      | 150      | 149 |
| 15      | 150      | 146      | 148      | 157 |
| 16      | 147      | 144      | 148      | 149 |
| 17      | 155      | 150      | 153      | 148 |
| 18      | 157      | 148      | 149      | 153 |
| 19      | 153      | 155      | 149      | 151 |
| 20      | 155      | 142      | 150      | 150 |
| 21      | 146      | 156      | 148      | 160 |
| 22      | 152      | 147      | 158      | 154 |
| 23      | 143      | 156      | 151      | 151 |
| 24      | 151      | 152      | 157      | 149 |
| 25      | 154      | 140      | 157      | 151 |
| 26      |          |          |          |     |

20 readings taken at regular intervals, ex. Every 15 minutes of production

Each reading consists of 4 samples of 4 separate parts. Hence sample size is 4





## STEPS IN ASSESSING PROCESS STABILITY



1. Collect Data
2. Calculate:  $\bar{X}$  & R for each sample

The screenshot shows the Minitab 'Row Statistics' dialog box. The 'Statistic' section has 'Mean' selected. The 'Input variables' field contains 'Sample1-Sample4'. The 'Store result in:' field is set to 'C5'. Other options like Sum, Standard deviation, Minimum, Maximum, Range, Median, Sum of squares, N total, N nonmissing, and N missing are also visible but not selected.



## SAMPLE DATA – STEEL ROD LENGTH EXAMPLE



|    | C1      | C2       | C3       | C4       | C5     | C6    |
|----|---------|----------|----------|----------|--------|-------|
|    | Sample1 | Sample 2 | Sample 3 | Sample 4 | Mean   | Range |
| 1  | 144     | 146      | 154      | 146      | 147.50 | 10    |
| 2  | 151     | 150      | 134      | 153      | 147.00 | 19    |
| 3  | 145     | 139      | 143      | 152      | 144.75 | 13    |
| 4  | 154     | 146      | 152      | 148      | 150.00 | 8     |
| 5  | 157     | 153      | 155      | 157      | 155.50 | 4     |
| 6  | 157     | 150      | 145      | 147      | 149.75 | 12    |
| 7  | 149     | 144      | 137      | 155      | 146.25 | 18    |
| 8  | 141     | 147      | 149      | 155      | 148.00 | 14    |
| 9  | 158     | 150      | 149      | 156      | 153.25 | 9     |
| 10 | 145     | 148      | 152      | 154      | 149.75 | 9     |
| 11 | 151     | 150      | 154      | 153      | 152.00 | 4     |
| 12 | 155     | 145      | 152      | 148      | 150.00 | 10    |
| 13 | 152     | 146      | 152      | 142      | 148.00 | 10    |
| 14 | 144     | 160      | 150      | 149      | 150.75 | 16    |
| 15 | 150     | 146      | 148      | 157      | 150.25 | 11    |
| 16 | 147     | 144      | 148      | 149      | 147.00 | 5     |
| 17 | 155     | 150      | 153      | 148      | 151.50 | 7     |
| 18 | 157     | 148      | 149      | 153      | 151.75 | 9     |
| 19 | 153     | 155      | 149      | 151      | 152.00 | 6     |
| 20 | 155     | 142      | 150      | 150      | 149.25 | 13    |
| 21 | 146     | 156      | 148      | 160      | 152.50 | 14    |
| 22 | 152     | 147      | 158      | 154      | 152.75 | 11    |
| 23 | 143     | 156      | 151      | 151      | 150.25 | 13    |
| 24 | 151     | 152      | 157      | 149      | 152.25 | 8     |
| 25 | 154     | 140      | 157      | 151      | 150.50 | 17    |

Calculate:  $\bar{X}$  & R for each sample

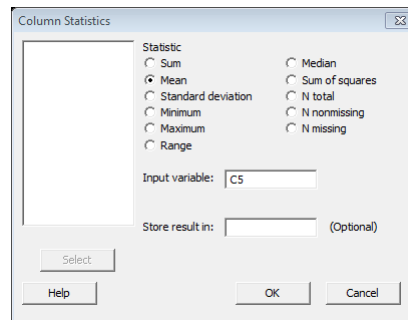
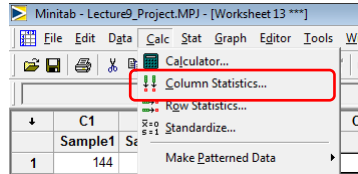




## STEPS IN ASSESSING PROCESS STABILITY



1. Collect Data
2. Calculate:  $\bar{X}$  & R for each sample
3. Calculate:
  - The Process Mean (Mean of Means)
  - The Mean Range (Average value of Ranges)

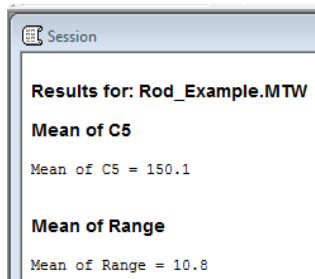


## SAMPLE DATA – STEEL ROD LENGTH EXAMPLE



|    | C1      | C2       | C3       | C4       | C5     | C6    |
|----|---------|----------|----------|----------|--------|-------|
|    | Sample1 | Sample 2 | Sample 3 | Sample 4 | Mean   | Range |
| 1  | 144     | 146      | 154      | 146      | 147.50 | 10    |
| 2  | 151     | 150      | 134      | 153      | 147.00 | 19    |
| 3  | 145     | 139      | 143      | 152      | 144.75 | 13    |
| 4  | 154     | 146      | 152      | 148      | 150.00 | 8     |
| 5  | 157     | 153      | 155      | 157      | 155.50 | 4     |
| 6  | 157     | 150      | 145      | 147      | 149.75 | 12    |
| 7  | 149     | 144      | 137      | 155      | 146.25 | 18    |
| 8  | 141     | 147      | 149      | 155      | 148.00 | 14    |
| 9  | 158     | 150      | 149      | 156      | 153.25 | 9     |
| 10 | 145     | 148      | 152      | 154      | 149.75 | 9     |
| 11 | 151     | 150      | 154      | 153      | 152.00 | 4     |
| 12 | 155     | 145      | 152      | 148      | 150.00 | 10    |
| 13 | 152     | 146      | 152      | 142      | 148.00 | 10    |
| 14 | 144     | 160      | 150      | 149      | 150.75 | 16    |
| 15 | 150     | 146      | 148      | 157      | 150.25 | 11    |
| 16 | 147     | 144      | 148      | 149      | 147.00 | 5     |
| 17 | 155     | 150      | 153      | 148      | 151.50 | 7     |
| 18 | 157     | 148      | 149      | 153      | 151.75 | 9     |
| 19 | 153     | 155      | 149      | 151      | 152.00 | 6     |
| 20 | 155     | 142      | 150      | 150      | 149.25 | 13    |
| 21 | 146     | 156      | 148      | 160      | 152.50 | 14    |
| 22 | 152     | 147      | 158      | 154      | 152.75 | 11    |
| 23 | 143     | 156      | 151      | 151      | 150.25 | 13    |
| 24 | 151     | 152      | 157      | 149      | 152.25 | 8     |
| 25 | 154     | 140      | 157      | 151      | 150.50 | 17    |

- Calculate:
- The Process Mean
  - The Mean Range





## STEPS IN ASSESSING PROCESS STABILITY



1. Collect Data
2. Calculate:  $\bar{X}$  & R for each sample
3. Calculate:
  - The Process Mean (Mean of Means)
  - The Mean Range (Average value of Ranges)
4. Plot these values on:
  - I. Mean Chart
  - II. Range Chart
 and examine the charts for any possible **Trends**



## MEAN & RANGE CHARTS



CONTROL CHART of the RANGE OF DUPLICATES

**Plot of Ranges**

**Plot of Means**

|   |  |   |   |
|---|--|---|---|
| Data previous chart(s)<br>$\bar{x}$ =<br>$UCL$ =<br>$LCL$ =<br>$R$ =<br>$\bar{R}$ = | Data this chart<br>$\bar{x}$ =<br>$UCL$ =<br>$LCL$ =<br>$R$ =<br>$\bar{R}$ = | Results:<br>Warning limit : $\bar{R} + 2\sigma$ =    +    -<br>Action limit : $\bar{R} + 3\sigma$ =    +    -<br>Remarks: | Verified by:<br>Date: _____<br>Signature: _____ |
|---|--|---|---|

|   |  |   |   |
|---|--|---|---|
| Data previous chart(s)<br>$\bar{x}$ =<br>$UCL$ =<br>$LCL$ =<br>$\bar{R}$ =<br>$\bar{R}$ = | Data this chart<br>$\bar{x}$ =<br>$UCL$ =<br>$LCL$ =<br>$R$ =<br>$\bar{R}$ = | Results:<br>Warning limits : $\bar{x} \pm 2\sigma$ =    ±    -    and<br>Action limits : $\bar{x} \pm 3\sigma$ =    ±    -    and<br>Remarks: | SOP: _____<br>Attribute: _____<br>Control Sample: _____<br>Serial No.: _____<br>Date: _____<br>Data previous chart(s)<br>$\bar{x}$ =<br>$UCL$ =<br>$LCL$ =<br>$\bar{R}$ =<br>$\bar{R}$ =<br>Data this chart<br>$\bar{x}$ =<br>$UCL$ =<br>$LCL$ =<br>$R$ =<br>$\bar{R}$ =<br>Results:<br>Warning limits : $\bar{x} \pm 2\sigma$ =    ±    -    and<br>Action limits : $\bar{x} \pm 3\sigma$ =    ±    -    and<br>Remarks: |
|---|--|---|---|

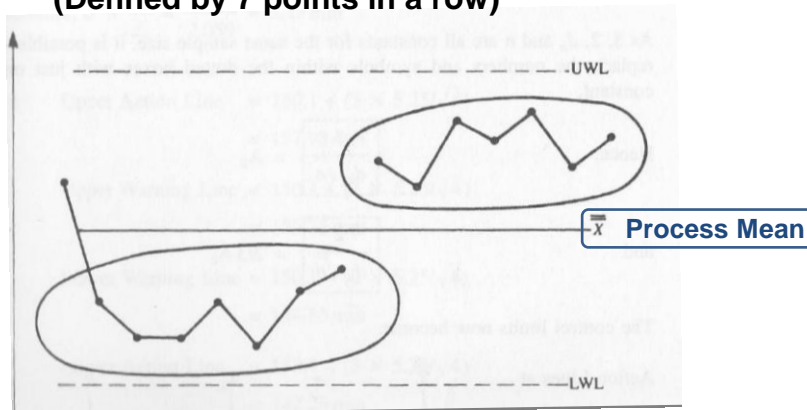




## PROCESS CONTROL - TRENDS



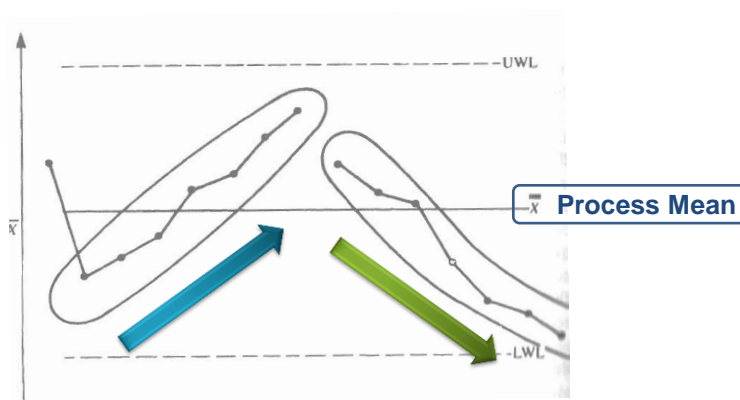
**A Run Above or Below the Process Mean Value  
(Defined by 7 points in a row)**



## PROCESS CONTROL - TRENDS

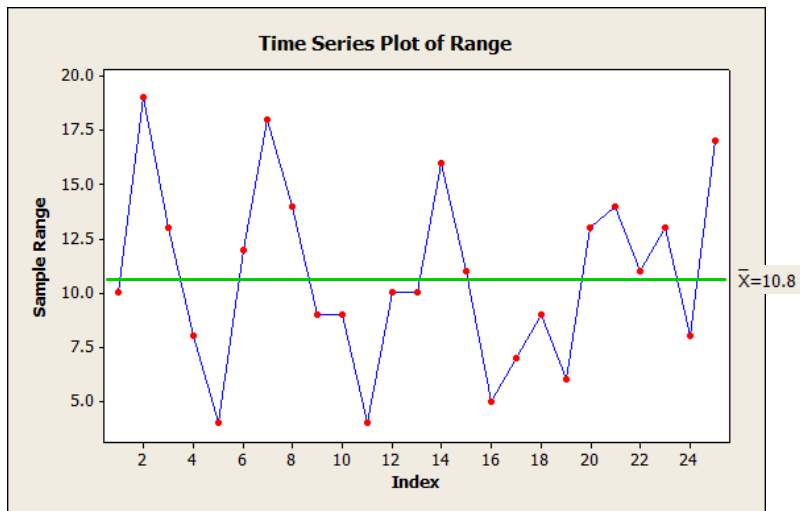


**A Rising or Falling Trend  
(Defined by 7 points in a row)**





**MEAN & RANGE CHARTS**  
**RANGES FOR ROD LENGTH EXAMPLE**



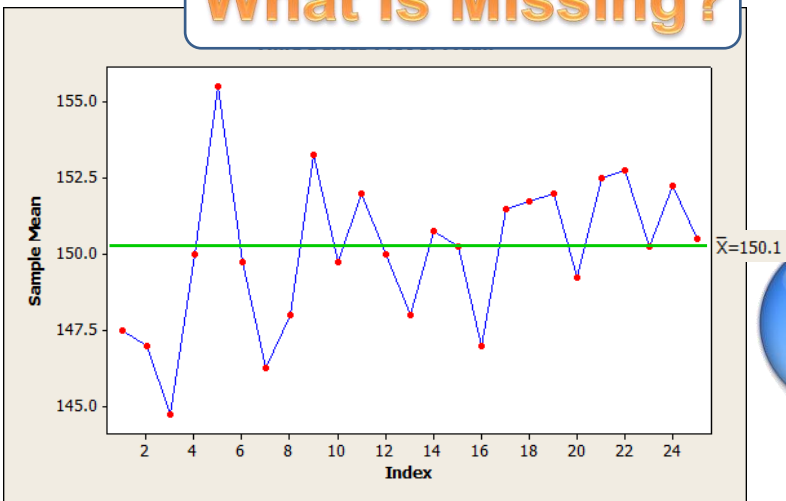
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**MEAN & RANGE CHARTS**  
**MEANS FOR ROD LENGTH EXAMPLE**



**What is Missing?**



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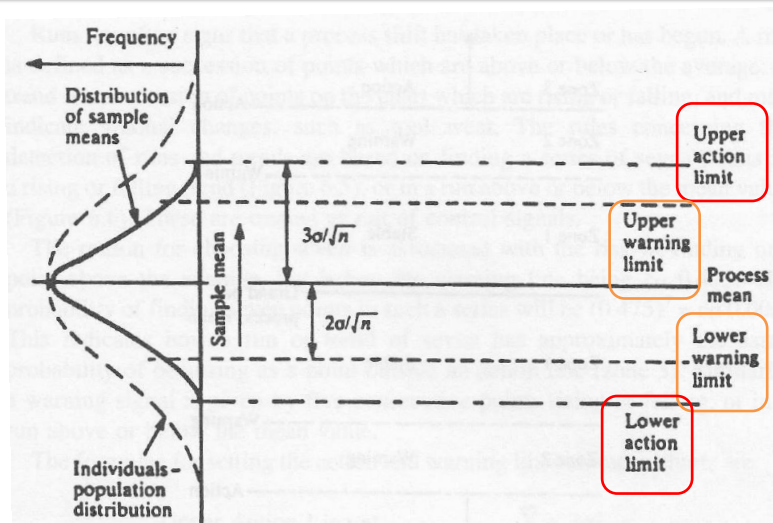
## STEPS IN ASSESSING PROCESS STABILITY



1. Collect Data
2. Calculate:  $\bar{X}$  & R for each sample
3. Calculate:
  - The Process Mean (Mean of Means)
  - The Mean Range (Average value of Ranges)
4. Plot these values and examine the charts for any possible **Trends**
5. Calculate the values and plot the control limits (**UCL & LCL**) on the charts



## CONTROL LIMITS





## ACTION AND WARNING LINES FOR MEAN CHARTS



Upper and Lower Control Limits (UCL & LCL)

| $\bar{X}$ -chart |                              | Control limit coefficients |               |             |       |
|------------------|------------------------------|----------------------------|---------------|-------------|-------|
| UCL              | $\bar{\bar{X}} + A_2\bar{R}$ | Sample size                | Average chart | Range chart |       |
|                  |                              | $n$                        | $A_2$         | $D_3$       | $D_4$ |
|                  |                              | 2                          | 1.880         | 0           | 3.267 |
|                  |                              | 3                          | 1.023         | 0           | 2.574 |
|                  |                              | 4                          | 0.729         | 0           | 2.282 |
| LCL              | $\bar{\bar{X}} - A_2\bar{R}$ | 5                          | 0.577         | 0           | 2.114 |

The Constant  $A_2$  is chosen dependent on the sample size



## ACTION AND WARNING LINES FOR RANGES CHARTS



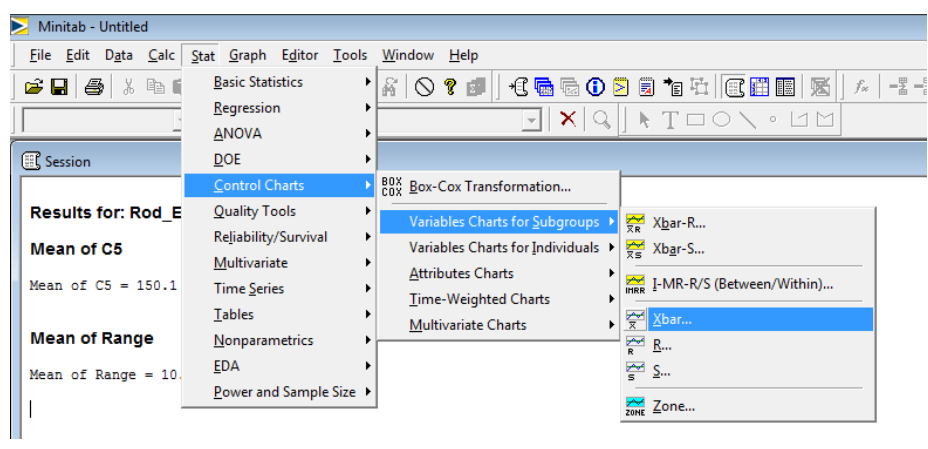
Upper and Lower Control Limits (UCL & LCL)

| R-chart |              | Control limit coefficients |               |             |       |
|---------|--------------|----------------------------|---------------|-------------|-------|
| UCL     | $D_4\bar{R}$ | Sample size                | Average chart | Range chart |       |
|         |              | $n$                        | $A_2$         | $D_3$       | $D_4$ |
|         |              | 2                          | 1.880         | 0           | 3.267 |
|         |              | 3                          | 1.023         | 0           | 2.574 |
|         |              | 4                          | 0.729         | 0           | 2.282 |
| LCL     | $D_3\bar{R}$ | 5                          | 0.577         | 0           | 2.114 |

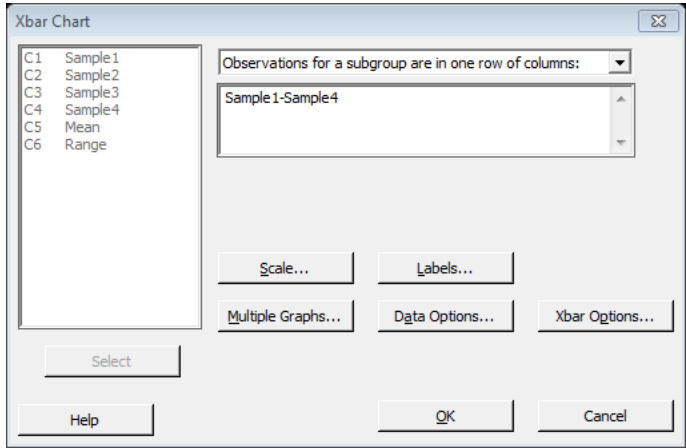




# MEAN PLOTS IN MINITAB

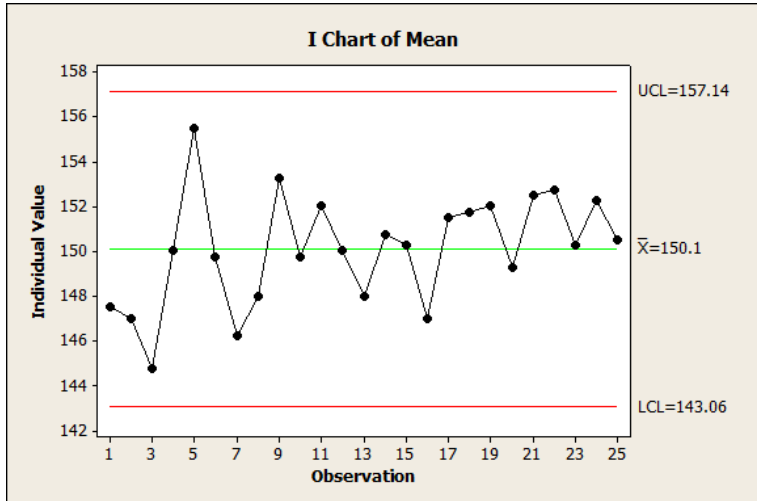


# MEAN PLOTS IN MINITAB





## MEAN & RANGE CHARTS MEANS FOR ROD LENGTH EXAMPLE



## PLOTS IN MINITAB



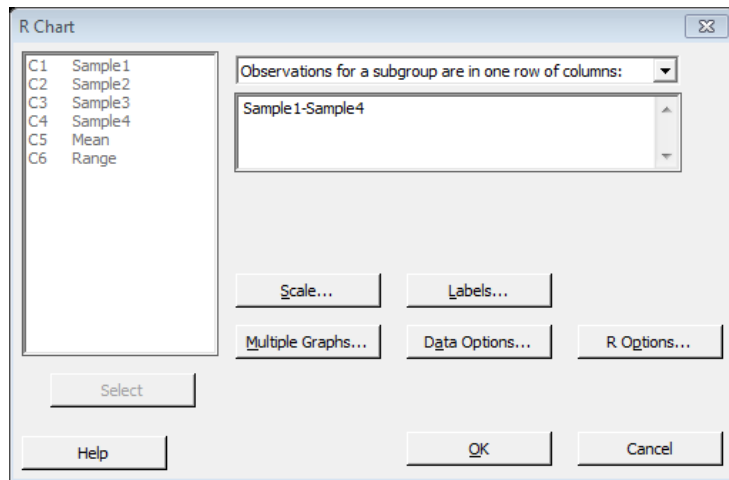
Minitab - Lecture9\_Project.MPJ - [Worksheet 13 \*\*\*]

|    | C1      | C2     | C5     | C6 | C7 | C8 | C9 | C10 | C11 |
|----|---------|--------|--------|----|----|----|----|-----|-----|
|    | Sample1 | Sample |        |    |    |    |    |     |     |
| 1  | 144     | 14     |        |    |    |    |    |     |     |
| 2  | 151     | 15     |        |    |    |    |    |     |     |
| 3  | 145     | 13     |        |    |    |    |    |     |     |
| 4  | 154     | 14     |        |    |    |    |    |     |     |
| 5  | 157     | 15     |        |    |    |    |    |     |     |
| 6  | 157     | 15     |        |    |    |    |    |     |     |
| 7  | 149     | 14     |        |    |    |    |    |     |     |
| 8  | 141     | 14     | 146.25 | 18 |    |    |    |     |     |
| 9  | 158     | 15     | 148.00 | 14 |    |    |    |     |     |
| 10 | 145     | 148    | 153.25 | 9  |    |    |    |     |     |
|    |         |        | 149.75 | 9  |    |    |    |     |     |

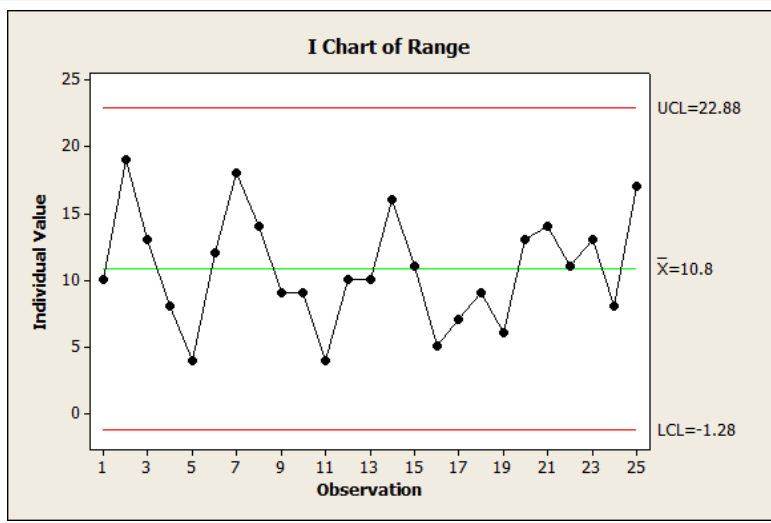




## PLOT OF RANGES



## MEAN & RANGE CHARTS RANGES FOR ROD LENGTH EXAMPLE





# PROCESS CAPABILITY



## PROCESS CAPABILITY

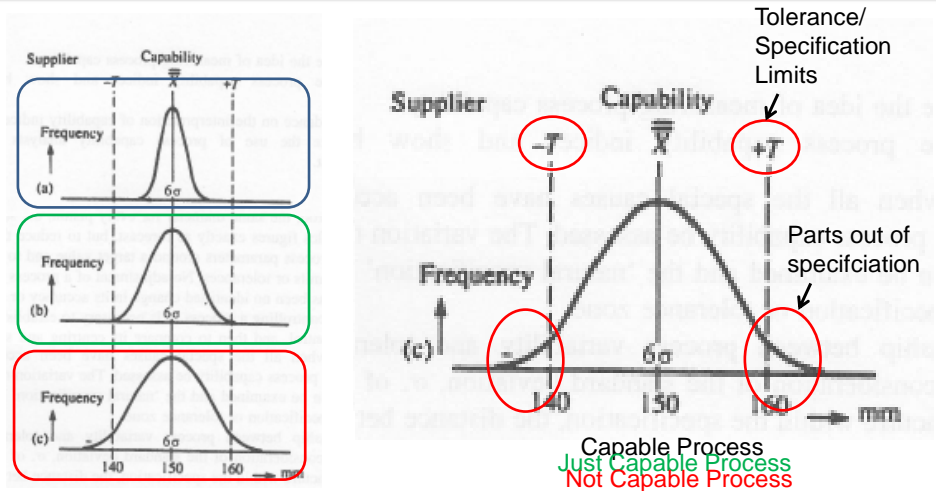


- Once the process is brought under statistical control by detecting and eliminating special causes of variation, its performance is predictable
- Its capability to meet customers' expectations can now be assessed.
- For this, we need measures which express the relationship between the **specified tolerance** (allowed deviation from the nominal) and the process variability.

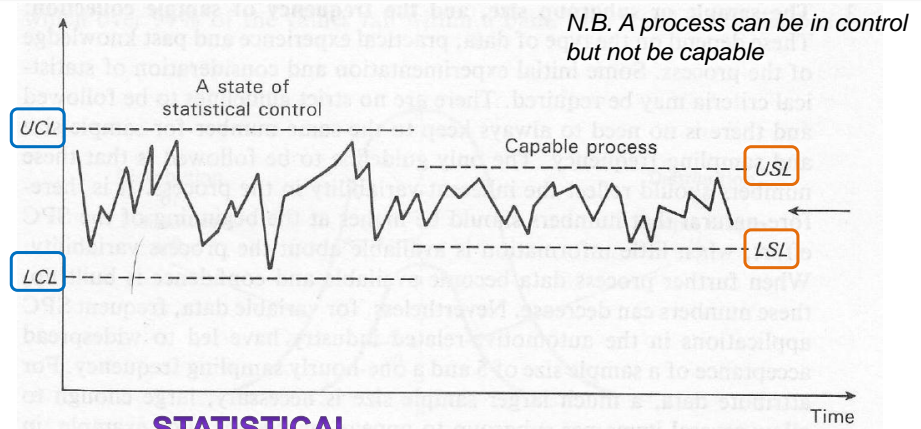




# PROCESS CAPABILITY



# PROCESS CAPABILITY



## STATISTICAL PROCESS CONTROL VS. PROCESS CAPABILITY CONTROL LIMITS VS. SPECIFICATION LIMITS





## PROCESS CAPABILITY



- **Process Capability Index:**

$$C_P = \frac{\text{Tolerance}}{6\sigma} = \frac{USL - LSL}{6\sigma}$$

- Or:

$$C_{pk} = \begin{cases} \frac{USL - \bar{X}}{3\sigma} \\ \frac{\bar{X} - LSL}{3\sigma} \end{cases}$$

**Note:**

The upper and lower specification limits (USL & LSL) are defined as the tolerances of the dimension.

- Where USL & LSL are the upper and lower specification limits

**Note:**

$C_{pk}$  is used for one sided or non-symmetrical tolerances.



## INTERPRETING CAPABILITY INDEXES



- $C_{pk} < 1$  Not Capable – Non Conforming Output
- $C_{pk} = 1$  Not Really Capable – Any Change will give a Non Conforming Output
- $C_{pk} = 1.33$  Not Satisfactory
- $C_{pk} = 1.5$  Not Satisfactory
- $C_{pk} = 1.67$  Promising
- $C_{pk} \geq 2$  High Level of Confidence





## PROCESS CAPABILITY - MINITAB



The screenshot shows the Minitab interface with the 'Stat' menu open. The path 'Stat > Process Capability > Capability Analysis > Normal...' is highlighted. The 'Worksheet 13 \*\*\*' window is visible in the background with the following data:

|   | C1      | C2       | C3       | C4       |
|---|---------|----------|----------|----------|
|   | Sample1 | Sample 2 | Sample 3 | Sample 4 |
| 1 | 144     | 146      | 154      |          |
| 2 | 151     | 150      | 134      |          |



## PROCESS CAPABILITY - MINITAB

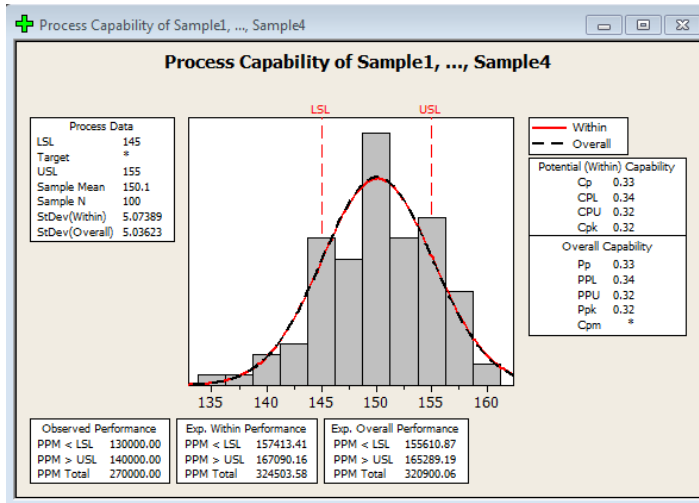


The dialog box 'Capability Analysis (Normal Distribution)' is shown. The 'Data are arranged as' section has 'Subgroups across rows of:' selected with 'Sample1-Sample4' in the dropdown. The 'Lower spec:' is set to 145 and 'Upper spec:' is set to 155. Both have 'Boundary' checkboxes that are unchecked. 'Historical mean:' and 'Historical standard deviation:' are set to empty fields with '(optional)' labels. Buttons for 'Box-Cox...', 'Estimate...', 'Options...', 'Storage...', 'OK', and 'Cancel' are visible.





## ROD LENGTH PROCESS CAPABILITY



## QUESTIONS





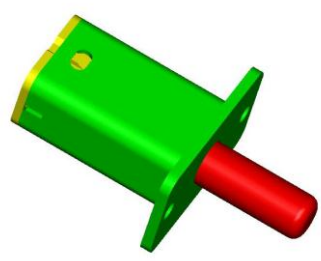
CoffeeBreak



# CASE STUDY

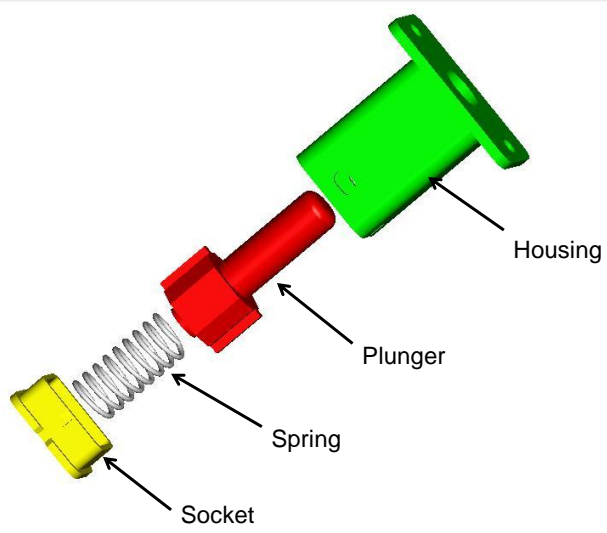


## FRIDGE DOOR LIGHT SWITCH





## CASE STUDY



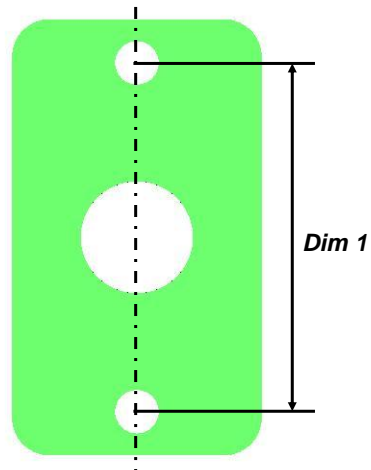
- Problem 1:**  
Housing locating holes are misaligned
- Problem 2:**  
Housing and Socket are not alligning properly



## PROBLEM 1



Housing Location Housing locating holes are misaligned



| Part Information            |                    |
|-----------------------------|--------------------|
| Part Material:              | PBT                |
| Process:                    | Injection Moulding |
| Injection Moulding Machine: | 2                  |
| <b>Specification:</b>       | <b>20+/-0.2</b>    |





## DATA ANALYSIS

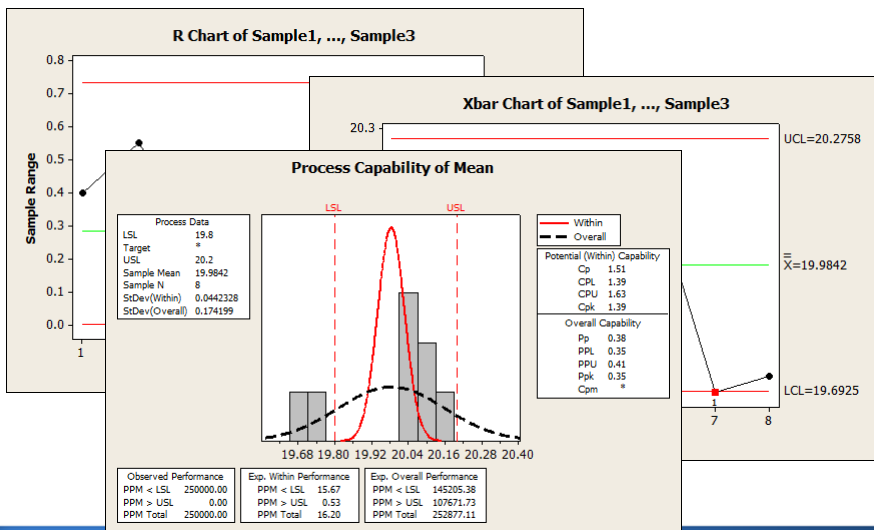


|    | C1      | C2      | C3      |
|----|---------|---------|---------|
|    | Sample1 | Sample2 | Sample3 |
| 1  | 20.30   | 20.20   | 19.90   |
| 2  | 19.75   | 20.10   | 20.30   |
| 3  | 19.86   | 20.05   | 20.13   |
| 4  | 20.10   | 20.20   | 19.98   |
| 5  | 19.90   | 20.20   | 20.10   |
| 6  | 20.10   | 19.90   | 20.30   |
| 7  | 19.65   | 19.70   | 19.72   |
| 8  | 19.67   | 19.78   | 19.73   |
| 9  |         |         |         |
| 10 |         |         |         |

1. Calculate:  $\bar{X}$  & R for each sample
2. Calculate:
  - $\bar{\bar{X}}$  The Process Mean (Mean of Means)
  - $\bar{R}$  The Mean Range (Average value of Ranges)
3. Plot these values and examine the charts for any possible **Trends**
4. Calculate the values and plot the control limits (UCL & LCL) on the charts
5. Check Process Capapbility

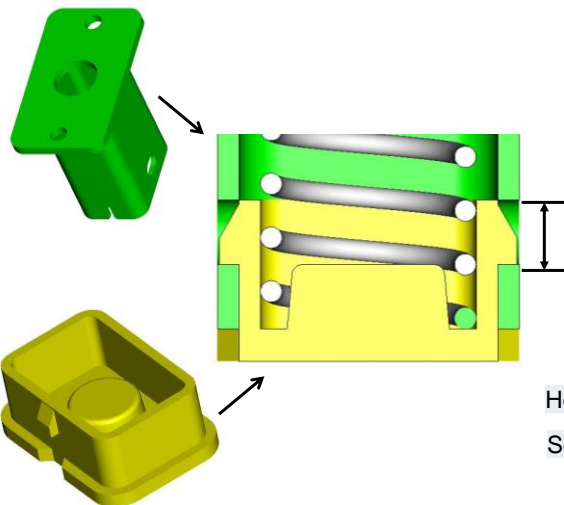


## DATA ANALYSIS: HOUSING





## PROBLEM 2



**Problem:**  
Interference between  
Housing Dim 2 and  
Socket Dim

Housing Specification:  $2.0+/-0.2$

Socket Specification:  $2.4+/-0.2$



## DATA ANALYSIS



| Housing_Dim2.MTW |         |         |         |    |    |    |    |    |
|------------------|---------|---------|---------|----|----|----|----|----|
| ↓                | C1      | C2      | C3      | C4 | C5 | C6 | C7 | C8 |
|                  | Sample1 | Sample2 | Sample3 |    |    |    |    |    |
| 1                | 2.10    | 2.12    | 2.19    |    |    |    |    |    |
| 2                | 2.13    | 2.08    | 2.05    |    |    |    |    |    |
| 3                | 2.15    | 2.06    | 2.07    |    |    |    |    |    |
| 4                | 2.18    | 2.22    | 2.17    |    |    |    |    |    |
| 5                | 2.15    | 2.20    | 2.25    |    |    |    |    |    |
| 6                | 2.13    | 2.08    | 2.05    |    |    |    |    |    |
| 7                | 2.23    | 2.10    | 2.19    |    |    |    |    |    |
| 8                | 2.25    | 2.15    | 2.21    |    |    |    |    |    |
| 9                |         |         |         |    |    |    |    |    |
| 10               |         |         |         |    |    |    |    |    |

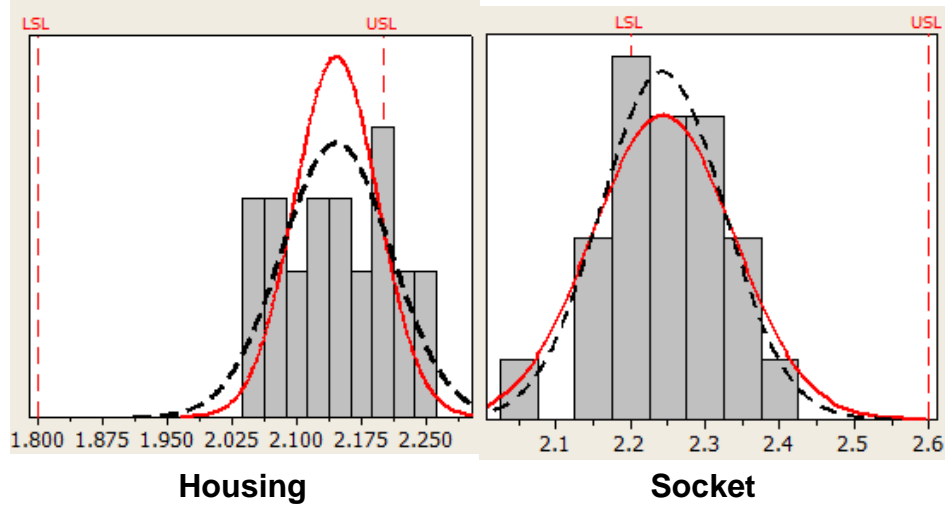
  

| Socket_Dim.MTW *** |         |         |         |    |
|--------------------|---------|---------|---------|----|
| ↓                  | C1      | C2      | C3      | C4 |
|                    | Sample1 | Sample2 | Sample3 |    |
| 1                  | 2.30    | 2.32    | 2.19    |    |
| 2                  | 2.32    | 2.18    | 2.05    |    |
| 3                  | 2.15    | 2.26    | 2.38    |    |
| 4                  | 2.18    | 2.35    | 2.20    |    |
| 5                  | 2.23    | 2.30    | 2.25    |    |
| 6                  | 2.13    | 2.15    | 2.35    |    |
| 7                  | 2.23    | 2.35    | 2.19    |    |
| 8                  | 2.25    | 2.31    | 2.21    |    |





# PROCESS CAPABILITY ANALYSIS



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# QUESTIONS



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## CONCLUSION LECTURE



- During this lecture we have seen:
  - **SPC by Variables**
  - **Process Capability**
  - **Practical Exercises using Minitab**

