UNIVERSITY OF MALTA
FACULTY OF SCIENCE
DEPARTMENT OF MATHEMATICS
MAT 1802 Mathematics for Engineers II
Problem Sheet 3

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(1) Determine a set of parametric equations of the line that contains the point $M_0(3, 5)$ and is parallel to the vector $\mathbf{a}$, where $\mathbf{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$. *{Ans:* $x = 3 + 2t,\ y = 5 + t$*}*

(2) Determine the equation of the line L that contains the point $M_0(1, -3)$ and is parallel to the line that contains the points $A(5, 1)$ and $B(-2, 4)$. *{Ans:* $3x + 7y + 18 = 0$*}*

(3) Determine an equation of the line that contains the point $M_0(5, 2)$ and is perpendicular to the vector $\mathbf{n}$, where $\mathbf{n} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$. *{Ans:* $-2x + 3y + 4 = 0$*}*

(4) Find an equation of the line L passing through the point $M_0(1, 2)$ and perpendicular to the line $2x - y - 3 = 0$. *{Ans:* $x + 2y - 5 = 0$*}*

(5) Find an equation of the line that contains the points $A(7, 3)$ and $B(5, 8)$ and determine also its parametric equations. *{Ans:* $5x + 2y - 41 = 0;\ x = 5 + 2t,\ y = 8 - 5t$*}*

(6) Determine an equation of the line that passes through $M_0(-2, 4)$ and whose angle of inclination is $\frac{\pi}{3}$. *{Ans:* $y = \sqrt{3}\,x + 4 + 2\sqrt{3}$*}*

(7) Find the intercept form of the equation of the line passing through the points $A(-2, -2)$ and $B(1, -8)$. *{Ans:* $\frac{x}{-3} + \frac{y}{-6} = 1$*}*

(8) Find the equation of a line L through the origin and perpendicular to the line $3x + 4y + 6 = 0$. *{Ans:* $4x - 3y = 0$*}*

(9) Find the acute angle formed by the two intersecting lines $4x - 2y - 1 = 0$ and $3x + y - 1 = 0$. *{Ans:* $\frac{\pi}{4}$*}*

(10) Find the normal form of the equation of the line $7x - 24y + 50 = 0$ and find the distance from the origin to the line. *{Ans:* $-\frac{7}{25}x + \frac{24}{25}y - 2 = 0;\ 2$*}*

(11) Find the deviation, and hence the distance, of the point $M_1(2, 1)$ from the line $3x + 4y - 5 = 0$. *{Ans: 1; 1}*

(12) Find an equation of the plane passing through the points $A(1, 2, 1)$, $B(0, -2, 3)$ and $C(-1, 2, -1)$. *{Ans:* $4x - 3y - 4z + 6 = 0$*}*

(13) Find an equation of the plane that contains the point $M_0(2, -4, 3)$ and is parallel to the plane determined by the vectors $\mathbf{a} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$. *{Ans:* $22x + y - 13z - 1 = 0$*}*

(14) Find the general form of the equation of the plane π passing through $M_0(2, 2, 1)$ and perpendicular to the vector $\overrightarrow{O\,\boldsymbol{M_0}}$. *{Ans:* $2x + 2y + z - 9 = 0$*}*

(15) Determine a unit vector perpendicular to the plane $3x + 6y - 2z + 10 = 0$. *{Ans:* $\frac{3}{7}\,\mathbf{e}_1 + \frac{6}{7}\,\mathbf{e}_2 - \frac{2}{7}\,\mathbf{e}_3$*}*

(16) Determine, if possible, the intercept form of the equation of the plane $3x + 4y + 6z - 12 = 0$. Sketch the plane. *{Ans:* $\frac{x}{4} + \frac{y}{3} + \frac{z}{2} = 1$*}*

(17) Find the equation of the plane π which passes through the point $M_0(6, 3, 4)$ and is parallel to the plane $2x + y - 2z + 8 = 0$. *{Ans:* $2x + y - 2z - 7 = 0$*}*

(18) Show that the planes $x - 3y + 2z + 6 = 0$ and $2x - 6y + 3z - 11 = 0$ are not parallel.

(19) Show that the planes $3x + 4y - 2z + 10 = 0$ and $2x + y + 5z - 13 = 0$ are perpendicular.

(20) Find the two angles that are formed when the planes $3x - 2y + 6z + 5 = 0$ and $3x + 5y - 8z + 3 = 0$ intersect. *{Ans:* $\frac{3\pi}{4}, \frac{\pi}{4}$*}*

(21) Find the distance of the point $M(1, 3, -2)$ from the plane $2x + 2y - z + 3 = 0$. *{Ans:* $\frac{13}{3}$*}*

(22) Determine the members of the family of parallel planes $3x - 12y + 4z + d = 0$ such that the distance of $M(1, 3, 2)$ from the planes is 2. *{Ans:* $d = -1, 51$*}*

(23) Show that the two planes $2x - y - z + 2 = 0$ and $6x - 3y - 3z + 10 = 0$ are parallel and find the distance between them. *{Ans:* $\frac{4}{3\sqrt{6}}$*}*

(24) Determine a set of parametric equations of the line that contains the point $M_0(4, -1, 6)$ and is parallel to the vector $\mathbf{d}$, where $\mathbf{d} = \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix}$. *{Ans:* $x = 4 + t,\ y = -1 + 5t,\ z = 6 + 3t$*}*

(25) Determine the equation of the line L passing through the points $A(4, 3, 4)$ and $B(-2, 5, 1)$. *{Ans:* $\frac{x-4}{-6} = \frac{y-3}{2} = \frac{z-4}{-3}$*}*

(26) Find the equation of the line defined by the two planes $2x + y - z + 1 = 0$ and $3x - y - z - 6 = 0$. *{Ans:* $\frac{x-1}{2} = \frac{y+3}{1} = \frac{z}{5}$ *}*