CLASSICAL MECHANICS

an introduction to

Newtonian and Lagrangian Methods

C J Camilleri

Motion of a Rigid Body about a Fixed Point

Inertial Frame $\quad X \equiv (Ox_i, \mathbf{e}_i)$

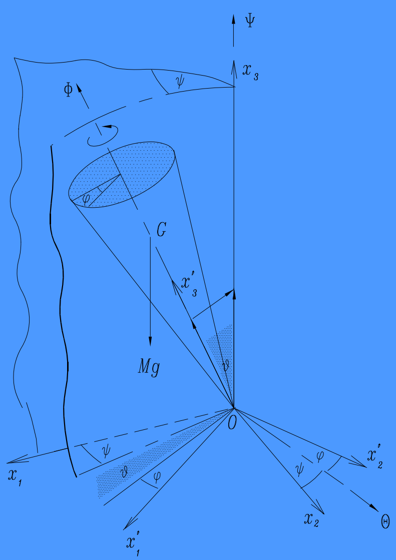
Rest Frame $\quad X' \equiv (Ox'_j, \mathbf{e}'_j)$

Euler's Angles $\quad (\psi,\ \theta,\ \phi)$

Principal Moments of Inertia $\quad (A,\ A,\ C)$

Angular velocity

$$\begin{aligned}\boldsymbol{\omega} &= (\dot{\theta}\sin\phi - \dot{\psi}\sin\theta\cos\phi)\,\mathbf{e}'_1 \\ &\quad + (\dot{\theta}\cos\phi + \dot{\psi}\sin\theta\sin\phi)\,\mathbf{e}'_2 \\ &\quad + (\dot{\phi} + \dot{\psi}\cos\theta)\,\mathbf{e}'_3\end{aligned}$$



The Spinning Top

Lagrangian

$$\mathcal{L} = \frac{1}{2}A(\dot{\theta}^2 + \dot{\psi}^2\sin^2\theta) + \frac{1}{2}C(\dot{\phi} + \dot{\psi}\cos\theta)^2 - Mgh\cos\theta$$

Electronic

Portable Document Format (PDF)

copy to

G M Borg

of

Msida, Malta

With best wishes

CLASSICAL MECHANICS

an introduction to

Newtonian and Lagrangian Methods

C J Camilleri
Department of Mathematics
University of Malta

Second Edition

July 16, 2004

First (Beta) Edition published in 1996
Second Edition in electronic-pdf format published in 2004
by C J Camilleri, Department of Mathematics
University of Malta, Tal-Qroqq, Msida, Malta

Typesetting and Page Layout Produced C J Camilleri.

The text was prepared by the author using LaTeX2.09 under Windows XP.
The diagrams were drawn by the author using AutoCad.

To my beloved wife Kate, who makes life a joy,
and my children
Anthony, Susan, David, Christopher

Contents

Preface

This book is based on lectures delivered to undergraduate students reading Mathematics, Physics or Engineering and is intended to cover a two-credit introductory course in Classical Mechanics. The motion of particles, rigid bodies and systems thereof are discussed. Both the Newtonian formulation with reference to Cartesian frames and the Lagrangian formulation with reference to generalized coordinates are discussed with a view to compare the two methods of approach.

These are alternative and equivalent formulations of the equations of motion both leading to differential equations which, when solved, describe the nature of the motion. In the Newtonian approach we consider the acceleration of the system under the action of the applied forces caused by external agencies. The resulting accelerated motion is governed by Newton's second law. In the Lagrangian approach we consider the kinetic and potential energies of the system, both of which are scalar functions and are therefore invariant under coordinate transformations. These transformations may be from Cartesian coordinates to generalized coordinates which may be chosen in one of many alternative ways that make the problem simpler to solve. The advantages of Lagrange's over Newton's equations are noted, particularly their independence on the choice of generalized coordinates. The book may be used as an introduction to any one of the two methods or both.

The student is required to have basic knowledge of vector algebra, matrix algebra and the calculus of ordinary functions of several variables including the chain rule for partial differentiation. He/She is also expected to be familiar with the suffix notation and the summation convention which are used wherever they result in more elegance and compactness. Besides it makes it easier for the student to make the transition to general relativity and mechanics of continua.

In rectangular Cartesian coordinates (x_i) a Latin suffix such as i has a range of values $(i = 1, 2, 3)$ and summation over this range of values is understood when a (dummy) suffix is repeated in any term. The Cartesian tensor and matrix notations are also employed whenever it is felt that they help to simplify the analysis as, for example, in rotating frames of reference, in moments of inertia and in Euler's equations.

In the Lagrangian method the index (suffix) notation is reserved for generalized coordinates (q_i) where a Latin index such as i has a range of values ($i = 1, 2, 3, \ldots, n$) and a repeated index in a term implies summation over this range of values. No distinction between covariant and contravariant indices is made as this is not necessary for the development of this work. Such distinction appears only in a few exercises which demonstrate the connection between Newton's and Lagrange's equations of motion. Here reference is made to relevant text on curvilinear coordinates and general tensors, but these exercises may be omitted unless the student is interested in these topics.

Chapters **1** and **2** deal with the basic concepts, definitions and laws used in classical mechanics to analyse the motion of a system relative to some frame of reference. Chapter **3** considers Cartesian frames of reference and the effect of their motion on the description of the motion of a system.

Chapter **4** is concerned with the mechanics of a single particle including its motion relative to a rotating frame. Lagrange's equations are introduced by a worked example and in some exercises by way of their verification for the case of the motion of a particle. These particular exercises could be omitted at this stage.

Chapter **5** considers the motion of a system of particles. This includes its motion relative to a frame which is itself translating and rotating with respect to an inertial frame. The two-particle system is discussed.

Chapter **6** discusses moments and products of inertia in preparation for the motion of rigid bodies which is covered by Newtonian formulation using vector methods in Chapters **7** and **8**.

In Chapter **9** we introduce the idea of virtual displacement and the method of virtual work to solve static systems. The motion of dynamic systems using Lagrange's and Hamilton's formulation is discussed in Chapters **10** and **11**, where generalized energy and momentum, ignorable coordinates and the Routhian function are defined. The configuration and phase spaces, Liouville's Theorem and Hamilton's Principle are also considered. The powerful methods of Lagrange are applied to impulsive motion and small oscillations.

There are many worked examples within the text and the exercises at the end of each section are intended to clarify the basic concepts introduced in that section. Moreover, some questions are intended to pose an extension of a concept which the student may develop for himself in preparation for the work that follows in later sections. In such cases hints or part solutions are given in the 'Answers and Notes' section at the end of the book. Starred questions are somewhat harder.

I have tried to provide as complete an index as possible and in some cases the number of pages indicated is rather large. An underlined page number indicates the most important reference to whatever is being indexed, its definition or its first appearance in the text shown in bold type.

I am thankful to Professor A. Buhagiar, Dr D. Buhagiar and Dr J. Muscat of the Department of Mathematics, University of Malta, who read and scrutinized the first and final drafts, made valid criticisms and suggestions, and many encouraging remarks.

I would also like to express my gratitude to the students who suffered through the many versions of the classroom editions of the text and for their help in checking the exercises.

Last but certainly not least, I wish to thank my wife for her unlimited patience, tolerance and continuous encouragement.

The book's faults, errors, and other imperfections are entirely my own particularly since I have typed the text and typeset it myself using LaTeX.

C J Camilleri

Department of Mathematics
University of Malta
July, 2004.

Published by C J Camilleri

This book provides a concise but complete treatment of classical mechanics.

It should meet the needs of undergraduate students in mathematics, science or engineering.

Both the Newtonian and the Lagrangian methods are discussed and compared. These are alternative and equivalent formulations of the equations of motion both leading to differential equations which, when solved, describe the nature of the motion. In the Newtonian approach we consider the acceleration of the system under the action of the applied forces caused by external agencies. The resulting accelerated motion is governed by Newton's second law. In the Lagrangian approach we consider the kinetic and potential energies of the system, both of which are scalar functions and are therefore invariant under coordinate transformations.

The suffix notation is used to denote Cartesian coordinates and the components of vectors and tensors in the 3-dimensional space. The notation is also used to denote generalized coordinates and components in the n-dimensional configuration space for a holonomic system with n degrees of freedom.

The employment Einstein's summation convention permits compact expressions and equations resulting in economy of written material by avoiding the repetition of similar terms.

CONTENTS

THE AUTHOR

C J CAMILLERI is professor emeritus in the Department of Mathematics at the University of Malta.