CSA4020

Multimedia Systems:
Adaptive Hypermedia Systems

Lecture 6:
Probabilistic Models
of
Information Retrieval

Disadvantages of Statistical Model

- Same doc in different collections can have different degrees of similarity to the same query

  A bit like deciding on whether a Honda Civic is a good purchase on its own merits vs. other members of family having a Merc.!

- Prefer to have impartial estimation of relevance to any query, irrespective of collection

  Provided by Boolean IR, but measure is too narrow (if all query terms in doc then relevant, otherwise not)

  We also know (from experience) that docs which do not necessarily contain the query terms may be relevant

More disadvantages

- A term which has a high *df* is given a low weight in statistical systems

  Could simply turn this into a high *probability* of observing the term...

  ... to *independently* determine the probability of relevance to a query of a document which contains that term

Binary Independence Retrieval Model

Fundamentals:

- Given a user query there is a set of documents which contains exactly the relevant documents and no other:

  the “ideal” answer set

- Given the ideal answer set, a query can be constructed that retrieves exactly this set

  Assumes that relevant documents are “clustered”, and that terms used adequately discriminate against non-relevant documents

- We do not know what are, in general, the properties of the ideal answer set

  All we know is that documents have terms which "capture" semantic meaning

- When user submits a query, "guess" what might be the ideal answer set

- Allow user to interact, to describe the probabilistic description of the ideal answer set (by marking docs as relevant/non-relevant)

Probabilistic Principle: Assumption

- Given a user query q and a document d_j in the collection:

  Estimate the probability that the user will find d_j relevant to q

- Rank documents in order of their probability of relevance to the query (Probability Ranking Principle)

- Model assumes that probability of relevance depends on q and document representations only

- Assumes that there *is* an ideal answer set!

- Assumes that terms are distributed differently in relevant and non-relevant documents

- Whether or not a document *x* is retrieved depends on:

  Pr(*rel*|*x*): the probability that *x* is relevant

  Pr(*nonrel*|*x*): ... *x* isn't relevant

- Document Ranking Function: document *x* will be retrieved if

$$a_2 \Pr(rel \mid x) \geq a_1 \Pr(nonrel \mid x)$$

  where a_2 is the cost of not retrieving a relevant document, and a_1 is the cost of retrieving a non-relevant document

$$g(x) = \frac{\Pr(rel \mid x)}{\Pr(nonrel \mid x)} \square \frac{a_1}{a_2} > 0$$

- If we knew Pr(*rel*|*x*) (or Pr(*nonrel*|*x*)), solution would be trivial, but...

- Use Bayes Theorem to rewrite Pr(*rel*|*x*):

$$\Pr(rel \mid x) = \frac{\Pr(x \mid rel)P(rel)}{\Pr(x)}$$

$\Pr(x)$: probability of observing x
$P(rel)$: *a priori* probability of relevance (ie, probability of observing a set of relevant documents)
$\Pr(x \mid rel)$: probability that x is in the given set of relevant docs

- Can do the same for $\Pr(nonrel \mid x)$

$$\Pr(nonrel \mid x) = \frac{\Pr(x \mid nonrel)P(nonrel)}{\Pr(x)}$$

- The document ranking function can be rewritten as:

$$\log g(x) = \log \frac{\Pr(x \mid rel)\Pr(rel)}{\Pr(x)} \frac{\Pr(x)}{\Pr(x \mid nonrel)\Pr(nonrel)}$$

and simplified as:

$$\log g(x) = \log \frac{\Pr(x \mid rel)}{\Pr(x \mid nonrel)} + \frac{\Pr(rel)}{\Pr(nonrel)}$$

- $\Pr(x \mid rel)$ and $\Pr(x \mid rel)$ are still unknown!

- We will replace them in terms of keywords in the document

- We assume that terms occur independently in relevant and nonrelevant documents...

$$\log g(x) = \sum_{i=1}^{t} \log \frac{\Pr(x_i \mid rel)}{\Pr(x_i \mid nonrel)} + C$$

- $\Pr(x_i \mid rel)$: probability that term x_i is present in a document randomly selected from the ideal answer set

- $\Pr(x_i \mid nonrel)$: probability that term x_i is present in a document randomly selected from outside the ideal answer set

- Considering document $D = \langle d_1, d_2, \ldots d_t \rangle$, where d_i is the weight of term i,

$$\log g(x) = \sum_{i=1}^{t} \frac{\Pr(x_i = d_i \mid rel)}{\Pr(x_i = d_i \mid nonrel)} + C$$

where $\Pr(x_i = d_i \mid rel)$ is the probability that a relevant document contains term x_i (similarly for $\Pr(x_i = d_i \mid nonrel)$)

- When $d_i = 0$ we want the contribution of term i to $g(x)$ to be 0:

$$\log g(x) = \sum_{i=1}^{t} \log \frac{\Pr(x_i = d_i \mid rel)}{\Pr(x_i = d_i \mid nonrel)} \frac{\Pr(x_i = 0 \mid nonrel)}{\Pr(x_i = 0 \mid rel)} + C$$

=

$$\log g(x) = \sum_{i=1}^{t} \log \frac{p_i(1 - q_i)}{q_i(1 - p_i)} + C$$

The term relevance weight of term x_i is:

$$tr_i = \log \frac{p_i(1 - q_i)}{q_i(1 - p_i)} = \log \frac{\Pr(x_i = d_i \mid rel)}{\Pr(x_i = d_i \mid nonrel)} \frac{\Pr(x_i = 0 \mid nonrel)}{\Pr(x_i = 0 \mid rel)}$$

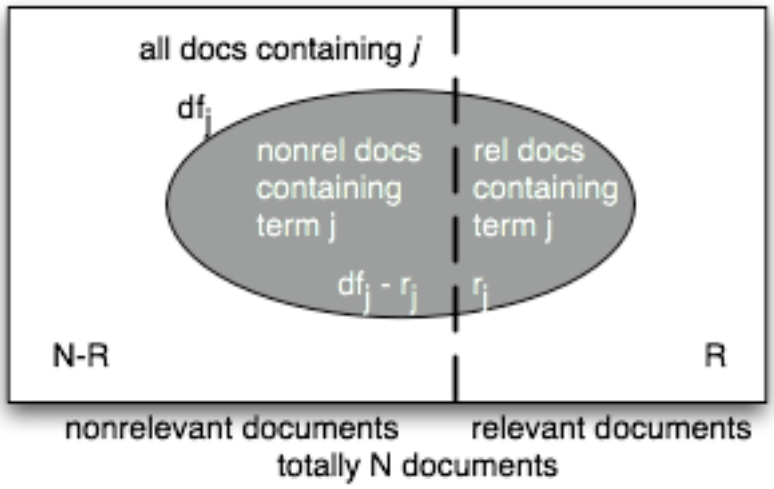
Weight of term i in document j is:

$$w_{ij} = tf_{ij} \cdot tr_i$$

Estimation of term occurrence probability

- Given a query, a document collection can be partitioned into a relevant and non-relevant set

- The importance of a term *j* is its discriminatory power in distinguishing between relevant and nonrelevant documents



- With *complete* information about the relevant and nonrelevant document sets we can estimate p_j and q_j:

$$p_j = r_j / R \qquad q_j = \frac{df_j - r_j}{N - R}$$

$$tr_j = \log \frac{r_j / R}{(df_j - r_j)/(N - R)} \frac{1 - (df_j - r_j)/(N - R)}{1 - r_j / R}$$

$$= \log \frac{r_j}{R - r_j} \frac{N - df_j - R + r_j}{df_j - r_j}$$

- Approximation: $tr_j = \log \frac{r_j}{R} \frac{N}{df_j}$

Term Occurrence Probability *Without* Relevance Information

- What do we do because we *don't* know r_j?

  $q_j = df_j / N$: since most docs are nonrelevant
  $p_j = 0.5$ (arbitrary)
  $tr_j = \log((N/df_j) \square 1)$: does this remind you of anything?

- Reminder... Ranking Function

$$\log g(x) = \sum_{i=1}^{t} \log \frac{p_i(1 \square q_i)}{q_i(1 \square p_i)} + C$$

where,

$$p_i = \Pr(x_i = d_i | rel)$$
$$q_i = \Pr(x_i = d_i | nonrel)$$

and d_i is the weight of term i