Minimum Rank of Real Symmetric Matrices

A real symmetric matrix represents an edge–weighted graph. The minimum rank of a simple graph G is the minimum rank among all real symmetric matrices whose zero-nonzero pattern of off diagonal entries is described by G. The diagonal entries are arbitrary and can be chosen to render the matrix non–singular. The minimum rank of a path P_n is $n-1$, the supremum for graphs of order n and that of the complete graph K_n is 1, the infimum. The minimum rank of a graph added to the maximum multipicity of its eigenvalues is the order of the graph. Only in 2008 has the minimum rank of the most popular classes of graphs been determined[1]. Determine other families of graphs whose minimum rank depend on graph parameters.

References:

[1] AIM Minimum Rank - Special Graphs Work Group, Zero forcing sets and the minimum rank of graphs, Linear Algebra and Appl. 428 7 (2008) 1628-1648.

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The Spectrum of the Laplacian

The degree–diagonal matrix D of a labelled graph has the vertex–degrees along its main diagonal. If A is the 0–1 adjacency matrix of a graph G, then $D - A$ is said to be the Laplacian Lap of G. By investigating the properties of the spectrum of Lap, discuss some of its useful applications.

Moore Graphs

A ρ–regular graph of diameter D and girth $2D + 1$ is said to be a Moore graph. Determine the number of vertices in terms of ρ and D. Show that ρ can take only four different values for $D = 2$. Discuss the properties of the possible Moore graphs of diameter two.

Number Patterns

The triangular array known as Pascal's triangle discovered by Ramon Llull (1232-1316) has fascinating latent patterns that may be investigated. There are other number patterns that were discovered by Bell and Euler among others. In this project, discover results in number theory based on visual designs or combinatorial schemes.

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