# MAT3105 Linear Algebra Project 2005/2006

*The purpose of this project is to investigate line graphs of trees and determine an eigenvector for the eigenvalue zero for those that are singular.*

A graph $G(\mathcal{V}, \mathcal{E})$ has a non-empty vertex set $\mathcal{V}$ and an edge set $\mathcal{E}$ consisting of pairs of distinct vertices. A tree is a graph without cycles. For the line graph $L_G$ of a graph $G$, the vertex set $\mathcal{V}(L_G) = |\mathcal{E}(G)|$ and two vertices in $L_G$ are adjacent if and only if the edges in $G$ have a common vertex. Thus the line graph of the star $K_{1,n-1}$ on $n$ vertices is the complete graph $K_{n-1}$.

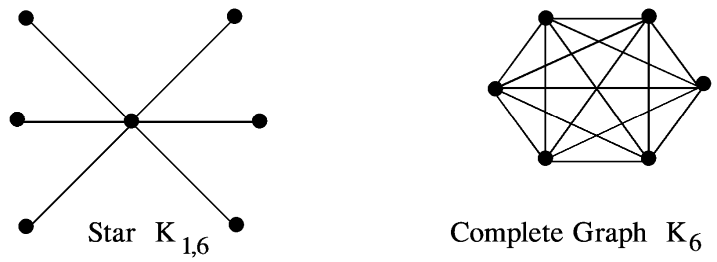


Figure 1: A star and its Line Graph

Also the line graph of a tree consists of adjacent complete graphs (cliques) and two cliques share at most one vertex.

For a graph $G$ on $n$ vertices, the adjacency matrix $\mathbf{A}(G) = \mathbf{A} = (a_{ij})$ of $G$ is the $n \times n$ symmetric matrix such that

$$a_{ij} = \begin{cases} 1 & \text{if } \{i, j\} \text{ is an edge of } G \\ 0 & \text{otherwise} \end{cases}$$

Figure 2 shows four trees:

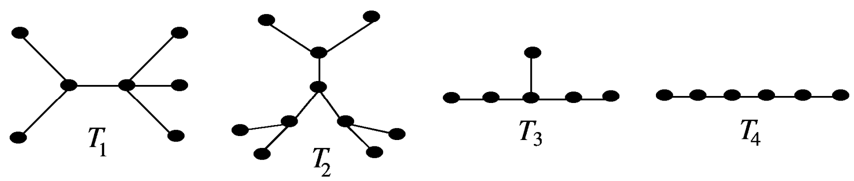


Figure 2: Trees

1. Label the vertices of each tree.

2. Call the Mathematica package <<DiscreteMath'Combinatorica'

3. Write down the adjacency lists of the trees and transform to the line graphs.

4. Determine the determinant $\det(L_T - v_i)$ of the adjacency matrix of each one-vertex deleted subgraph of $L_T$'

5. By using the 'Drop ' command or otherwise, find the adjugate $adjug(L_T)$ of $L_T$.

It is established in [1] that the nullity of the line graph of a tree is at most one.

1. Why are the columns of $adjug(L_T)$ eigenvectors of $A\,(L_T)$ for a singular line graph of a tree?

2. If each one-vertex deleted subgraph of the line graph of a tree is singular, why is $L_T$ non-singular?

With reference to the four trees $T_1,\ T_2,\ T_3,\ T_4$ above, discuss the relevance of the one-vertex deleted subgraphs of the line graphs $\{L_T - v_i\}$, to the nullspace of the adjacency matrix of $L_T$.

**Reference**


[1]

Sciriha, I., On Singular Line Graphs of Trees, Congressus Numeratium, 135 (1998) 73-91.