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THE TWO CLASSES OF SINGULAR LINE GRAPHS OF TREES

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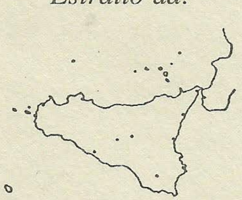

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THE TWO CLASSES OF SINGULAR LINE GRAPHS OF TREES*

IRENE SCIRIHA**

Abstract. *A graph G is singular if its adjacency matrix $A(G)$ is singular. We consider the multiplicity of the eigenvalue 0 for line graphs L_G and show that it is at most one for L_T where T is a tree. Moreover, if L_T is singular, then T is shown to be even. Furthermore, the Polynomial Reconstruction Conjecture, a variant of Ulam's Reconstruction Conjecture, is shown to be true for singular line graphs of trees. The analysis gives rise to a partition of singular line graphs of trees into two classes.*

1. Introduction.

A graph $G(\mathcal{V}, \mathcal{E})$ has a set $\mathcal{V} = \{v_1, v_2, \ldots, v_n\}$ of n vertices and an edge set $\mathcal{E}$ of m edges. Each edge joins a pair of distinct vertices. A **line graph** L_G of a root graph G has m vertices. Two vertices are adjacent in L_G if and only if the corresponding edges in G have a common vertex. A **Krausz partition** of a line graph L_G, is the set of

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cliques (maximal complete subgraphs) such that every edge of L_G is in exactly one clique and every vertex of L_G is in exactly two cliques.

The **adjacency matrix** $\mathbf{A} = (a_{ij})$ of a graph G, is the $n \times n$ symmetrical matrix such that $a_{ij} = 1$ when v_i is adjacent to v_j, and $a_{ij} = 0$, otherwise. The real numbers λ for which there exist non-zero values of $\mathbf{x}$ satisfying the equation

$$\text{(1)} \qquad \mathbf{Ax} = \lambda\mathbf{x}.$$

are called the **eigenvalues** of G. For a particular eigenvalue λ, $\mathbf{x}$ is called an **eigenvector** in the eigenspace of λ. The characteristic polynomial of A is denoted by $\phi(G, \lambda)$ $(= \phi(G))$ and if I_n denotes the identity matrix, then

$$\text{(2)} \qquad \phi(G, \lambda) = \operatorname{Det}(\lambda I_n - A) = \prod_{i=1}^{n}(\lambda - \lambda_i)$$

is a polynomial $\sum_{i=0}^{n} q_i\lambda^i$ with integer coefficients q_i. The values $\lambda_1, \lambda_2, \ldots, \lambda_n$ are, therefore, the eigenvalues of G and form the **spectrum**, $Sp(G)$, of G.

If G is singular, then at least one of the eigenvalues of $\mathbf{A}$ is zero. The kernel of the linear transformation $\mathbf{A}$ is called the **zero-space** or **nullspace** of $\mathbf{A}$. An eigenvector $\mathbf{x_0}$ in the zero-space of $\mathbf{A}$ is called a **kernel eigenvector** and satisfies the equation

$$\text{(3)} \qquad \mathbf{Ax} = \mathbf{0}$$

Since $\mathbf{A}$ is symmetrical, the algebraic multiplicity of an eigenvalue equals its geometric multiplicity and this common value, for the eigenvalue zero, is the **nullity** of $\mathbf{A}$, denoted by $\eta(G)$.

In section 2, we establish two properties particular to singular line graphs of trees. The first is that the nullity is one; the second is that the order of the tree is even.

In section 3, we introduce Ulam's Reconstruction Conjecture. A variation of Kelly's and Ulam's reconstruction conjectures [2, 3], posed by I. Gutman and D. M. Cvetkovic (1974), is to reconstruct the characteristic polynomial of a graph from the **p-deck**, the deck of the characteristic polynomials of the one-vertex-deleted subgraphs [4]. Instead of the **p-deck**, we can start with the **s-deck**, the deck of the spectra of the one-vertex-deleted subgraphs, from which the p-deck can be produced. A. Schwenk discusses this conjecture and refers to it as problem D [6]. Positive results were established by S. Simic in connection with the class of connected graphs having the smallest eigenvalue in the p-deck bounded below by -2 [10]. These include the set of line graphs of trees $\{L_T\}$. In section 4, we give a new proof of the polynomial reconstruction conjecture (problem D) for singular line graphs of trees. The proof also shows that the set of singular line graphs of trees partitions itself into two disjoint classes, determined by the presence or otherwise of a zero entry in the kernel eigenvector. We show this classification in section 5. In the final section we pose two problems on the structure of singular line graphs of trees.

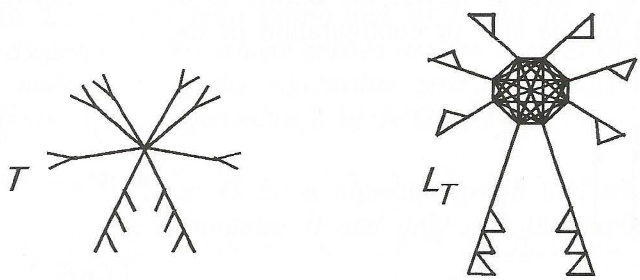


Figure 1 - A Tree T and its line graph L_T.

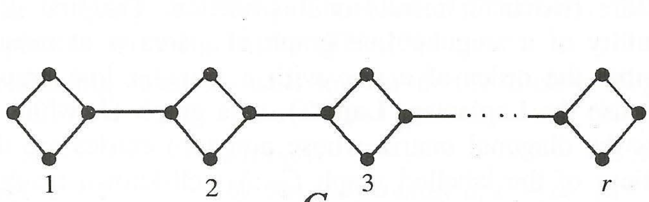


Figure 2 - The Graph G_r

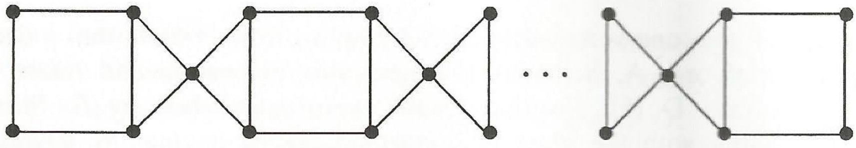


Figure 3 - The Graph $L(G_r)$) with $\eta(L(G_r))=r+1$.

2. Trees with Singular Line Graphs.

The graph G_r, shown in Fig.2, has a line graph (Fig.3) whose nullity increases with r. The same holds for the nullity of the line graph of pK_2 which is p. Thus the nullity of a line graph may assume any positive integer and $\forall p \in \mathbf{N}, \exists\ G$ with $\eta(L_G) = p$. We show that if G is a tree, however, the nullity is bounded above and does not depend on the size or configuration of the tree.

Figure 4 - The Graph pK_2 and its line graph.

There are two main results in this section. The first is the result that the nullity of a singular line graph of a tree is at most one. The second is that the order of a tree with a singular line graph is even. The proofs use the Laplacian, $\mathbf{Lap}(G)$, of a graph G, which is $\mathbf{D} - \mathbf{A}$, where $\mathbf{D}$ is the diagonal matrix whose non-zero entries are the degrees of the vertices of the labelled graph G. A well-known result regarding the Laplacian, which we have occasion to use in some proofs, is

The Matrix Tree Theorem: Let G be a labelled graph. All the cofactors of $\mathbf{Lap}(G)$ are equal and their common value is the number of spanning trees in G.

Remark For a tree, the number of spanning trees is one. Thus each entry of the adjoint of $\mathbf{Lap}(T)$ is one.

We shall now prove a number of lemmas that lead to the result that the nullity of the line graph of a tree is at most one.

LEMMA 2.1. [Grone, Merris and Sunder [5]] *For a bipartite graph G with adjacency matrix, $\mathbf{A}$, the Laplacian, $\mathbf{D} - \mathbf{A}$, of $\mathbf{A}$, has the same eigenvalues as the matrix $\mathbf{A} + \mathbf{D}$.* □

Remark 2.2. We note that this result holds for a tree, which is necessarily bipartite.

The following lemma follows from the well known relation between the characteristic polynomial of a line graph and that of the root graph [1];

$$\phi(L_G, \lambda) = (\lambda + 2)^{m-n}\phi(\mathbf{A}(G) + \mathbf{D}_G, \lambda + 2).$$

LEMMA 2.2. *Let G be a graph and let L_G be its line graph. Let $\mathbf{D}_G$ be the diagonal matrix whose entries are the degrees of the vertices of G. The multiplicity of the eigenvalue zero in $Sp(L_G)$ is equal to the multiplicity of the eigenvalue 2 of $\mathbf{A}(G) + \mathbf{D}_G$.* □

COROLLARY 2.1. *Let G be a bipartite graph and let L_G be its line graph. Then L_G is singular if and only if 2 is an eigenvalue of* $\mathbf{Lap}(G)$.

Proof. This follows from Lemmas 2.1 and 2.2. □

Remark 2.3. A characterisation of trees with **singular line graphs** would settle a query raised by Grone *et al* , namely: "Which trees have a Laplacian with eigenvalue 2?". [5]

The following result is well known in the theory of polynomial rings.

LEMMA 2.3. *Let $f(x)$ be the polynomial $a_n x^n + a_{n-1}x^{n-1} + \ldots + a_0$ with integer coefficients a_i. If $x = \frac{p}{q}$ is a rational root (reduced to its lowest terms) of the equation $f(x) = 0$, then p divides a_0 and q divides a_n.* □

COROLLARY 2.2. *Let G be a graph such that $\mathbf{A}$ is unimodular. If λ is a rational eigenvalue in the spectrum of G, then λ is the integer 1 or -1.*

Proof. We recall that a matrix is unimodular if it is an integer matrix whose determinant is ± 1. If the characteristic polynomial $\phi(G, \lambda)$ of $\mathbf{A}$ is $a_n\lambda^n + a_{n-1}\lambda^{n-1} + \ldots + a_0$, then $a_n = 1$ for all G and $a_0 = \pm 1$ since $\mathbf{A}$ is unimodular. Thus by Lemma 2.3, the stated result follows. □

LEMMA 2.4. *If T is a tree on n vertices such that L_T is singular and $\exists \mathbf{x} \neq 0$ s.t. Lap $\mathbf{x} = 2\mathbf{x}$ then $\mathbf{x}$ has no zero entries.*

Proof. Let $\mathbf{x}$ be an eigenvector of $\mathbf{Lap}(T)$ corresponding to an eigenvalue 2. Suppose that some entry of $\mathbf{x}$ is zero and let T be labelled so that the vertex corresponding to this zero entry is the nth vertex v_n of degree $\rho(v_n)$. There are $\rho(v_n)$ subgraphs (each of which is a tree T_i) coalesced at v_n. As shown in Fig.4, this means that the trees $T_1, T_2, \ldots, T_{\rho(v_n)}$ share the common vertex v_n.

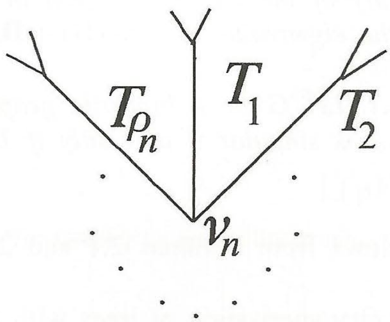


Figure 5 - A tree T with the nth entry of $\mathbf{x}$, s.t. $\mathbf{Lap}(T)(\mathbf{x})=2\mathbf{x}$, being zero.

For a labelling of T

$$\mathbf{Lap}(T) = \begin{pmatrix} \mathbf{L}_1 & 0 & 0 & \ldots & 0 & * \\ 0 & \mathbf{L}_2 & 0 & \ldots & 0 & * \\ 0 & & \mathbf{L}_3 & \ldots & 0 & * \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & \mathbf{L}_{\rho(v_n)} & * \\ ** & * & * & \ldots & * & \rho(v_n) \end{pmatrix}$$

where $\mathbf{L}_i$ corresponds to the ith subgraph $T_i - v_n$ which is T_i with the vertex v_n deleted. Let $\mathbf{x}$ be the vector $(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{\rho(v_n)}, 0)$ where

$\mathbf{x}_i$, itself a vector, corresponds to $\mathbf{L}_i$. The relation $\mathbf{Lap}(T)(\mathbf{x}) = 2\mathbf{x}$ implies that $\mathbf{L}_i(\mathbf{x}_i) = 2\mathbf{x}_i$ for each $\mathbf{L}_i$. Thus there exists $\mathbf{L}_k$ with $\mathbf{x}_k \neq 0$ and eigenvalue 2. Now $\mathbf{L}_k$ differs from $\mathbf{Lap}(T_k - v_n)$ in the diagonal entry corresponding to $v_j \in \mathcal{V}(T_k)$, where v_j is the vertex of T_k which in T is a neighbouring vertex of v_n. But the determinant $\mathrm{Det}(\mathbf{L}_k) = \mathrm{Det}(\mathbf{Lap}(T_k - v_n)) + \mathrm{Det}(\mathbf{L}_{jj})$ where $\mathbf{L}_{jj}$ is the submatrix of $\mathbf{L}_k$ obtained by deleting the jth row and column of $\mathbf{L}_k$. The matrix $\mathbf{L}_{jj}$ is the same as that obtained by deleting the jth row and column of $\mathbf{Lap}(T_k - v_n)$ so that by the Matrix Tree Theorem, since $T_k - v_n$ has only one spanning tree, $\mathrm{Det}(\mathbf{L}_{jj}) = 1$. Also since all Laplacians are singular, the term $\mathrm{Det}(\mathbf{Lap}(T_k - v_n))$ is zero. Thus the eigenvalue 2 satisfies $\mathbf{L}_k\mathbf{x}_k = \mu\mathbf{x}_k$ and is a root of the equation $\mathrm{Det}(\lambda\mathbf{I}_p - \mathbf{L}_k) = \lambda^p + b_{p-1}\lambda^{p-1} + \ldots + b_1\lambda \pm 1 = 0$, where $\mathbf{L}_k$ is of order $p \times p$. By Lemma 2.3, $\mu = \pm 1$ is the only rational root, a contradiction.

We now show that each entry of an eigenvector of $\mathbf{D}_T + \mathbf{A}(T)$ corresponding to the eigenvalue 2 is also non-zero. By Lemma 2.1, $\mathbf{D}_T + \mathbf{A}(T)$ has the same eigenvalues as $\mathbf{Lap}(T)$. Let the tree T be the bipartite graph $(\mathcal{V}_1, \mathcal{V}_2, \mathcal{E})$, labelled such that the top rows of $\mathbf{A}(T)$ correspond to the vertices of set $\mathcal{V}_1$. Let $\mathbf{P}$ be the diagonal matrix such that the non-zero entries corresponding to the vertices $\mathcal{V}_1$ are 1 and those corresponding to the vertices $\mathcal{V}_2$ are -1. Then $\mathbf{P}^{-1}(\mathbf{Lap}(T))\mathbf{P}$ is $\mathbf{D}_T + \mathbf{A}(T)$. Also

$$(\mathbf{D}_T + \mathbf{A}(T)) = \left(\begin{array}{c|c} \mathbf{D}_1 & \mathbf{C} \\ \hline \mathbf{C}^t & \mathbf{D}_2 \end{array}\right)$$

where $\mathbf{D}_1$ and $\mathbf{D}_2$ are degree diagonal matrices corresponding to $\mathcal{V}_1$ and $\mathcal{V}_2$ respectively and $\mathbf{C}$, $\mathbf{C}^t$ describe the edges between the two subsets of vertices in the tree T. If $(u, v)^t$ is a conformal partition of $\mathbf{x}$ and $\mathbf{Lap}(T)(\mathbf{x}) = \mu\mathbf{x}$, and $\mathbf{Lap}(T)\begin{pmatrix}\mathbf{u}\\ \mathbf{v}\end{pmatrix} = 2\begin{pmatrix}\mathbf{u}\\ \mathbf{v}\end{pmatrix}$,

then $(\mathbf{D}_T + \mathbf{A}(T))\begin{pmatrix}\mathbf{u}\\ -\mathbf{v}\end{pmatrix} = 2\begin{pmatrix}\mathbf{u}\\ -\mathbf{v}\end{pmatrix}$.

So each entry of an eigenvector of $\mathbf{D}_T + \mathbf{A}(T)$, corresponding to eigenvalue 2, is non-zero. □

THEOREM 2.1. $\eta(L_T) = 1$.

Proof. Since the dimension of the 2-space of $\mathbf{D}_T + \mathbf{A}(T)$ is one, the dimension of the nullspace of L_T is at most one. □

THEOREM 2.2. *If L_T is singular, then T is even.*

Proof. The characteristic polynomial $\phi(\mathbf{Lap}(T), \mu)$ of $\mathbf{Lap}(T)$ is $\mathrm{Det}(\mu I - \mathbf{Lap}) = \mu(\mu - \mu_2)...(\mu - \mu_m)$.

The absolute value of the coefficient of μ in $\phi(\mathbf{Lap}(T), \mu)$ is the trace of the adjoint of $\mathbf{Lap}(T)$. By the Matrix Tree Theorem each entry of the adjoint of the Laplacian is the number of spanning trees, which is one for a tree. Thus, if T has n vertices, then the coefficient of μ in $\phi(\mathbf{Lap}(T), \mu)$ is $\pm n$.

Since the nullity of the Laplacian of a connected graph is one, the coefficient of μ in $\phi(\mathbf{Lap}(T), \mu)$ is also numerically equal to the product of its non-zero eigenvalues of $\mathbf{Lap}(T)$. But by Corollary 2.1, for a singular line graph, L_G, 2 is an eigenvalue of $\mathbf{Lap}(G)$. So it follows that 2 divides n. □

COROLLARY 2.3. *The line graph of an odd tree is non-singular.* □

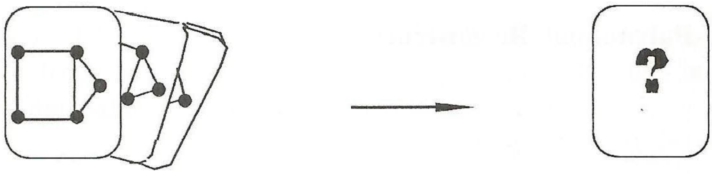
Remark 2.4. The converse is false. A counter example is $K_{1,3}$ whose line graph is K_3 which is non-singular.

3. Ulam's Reconstruction Conjecture.

Ulam's Reconstruction Conjecture: *Every graph with at least* 3 *vertices is reconstructible.*

Equivalently, for $n \geq 3$, given a deck $\mathcal{D}$ of n cards, each showing a subgraph $G - v$ as v runs through the n vertices of G, the graph G can be recovered. This problem has been solved for various classes of graphs but is still open in general. For regular graphs it is trivially true as can be seen from the example in Fig 6.

From the deck $\mathcal{D}$ we deduce that $n = 6$. Also the number of edges in each card is 6 so that the same number of edges are deleted with each vertex. Thus the parent graph G is regular. It is thus clear that regularity is recognisable from $\mathcal{D}$. To recover G, it suffices to add



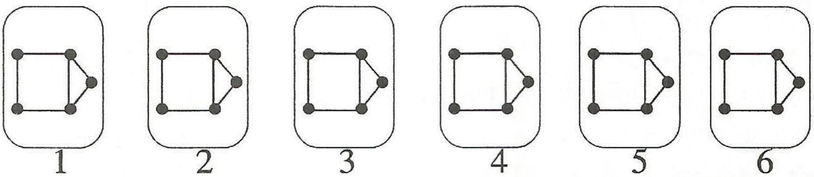


Figure 6 - The deck of a regular graph.

a vertex to one of the subgraphs and join it to those vertices in the subgraph which have the minimum degree.

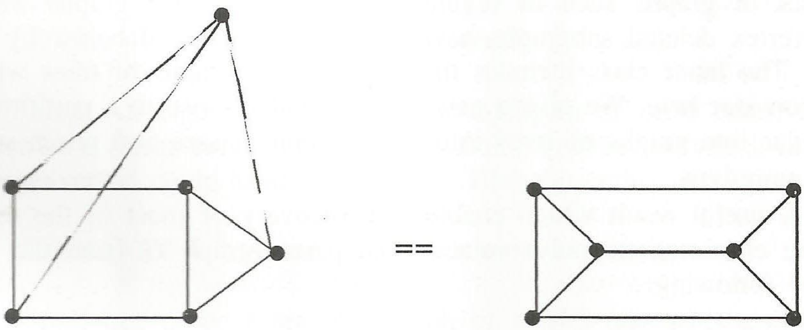


Figure 7 - Reconstruction of a regular graph.

Hence for **regular graphs** the conjecture is true. The case for regular graphs is very simple but the problem has proved to be very difficult for the arbitrary graph and is still open after about half a century of history.

4. The Polynomial Reconstruction Conjecture.

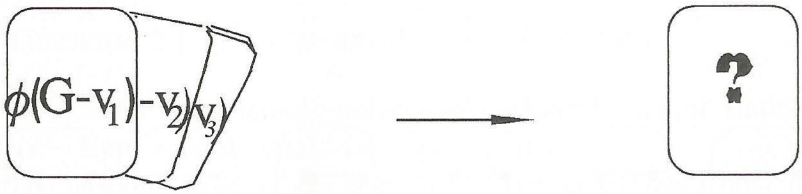


Figure 8 - The Polynomial Deck of a graph G.

The **Polynomial Reconstruction Conjecture** is a variant of Ulam's Reconstruction Conjecture. It states that: *Every graph with at least* 3 *vertices is polynomial reconstructible.*

Equivalently, for $n \geq 3$, given a p-deck $\mathcal{PD}$ of n cards, each showing a characteristic polynomial $\phi(G-v)$ as v runs through the n vertices of G, the characteristic polynomial $\phi(G)$ can be recovered. This problem is still open in general but has been solved for some classes of graphs such as regular graphs [4] and for graphs whose one-vertex deleted subgraphs have eigenvalues bounded below by -2 [10]. The latter class includes the singular line graphs of trees which we consider here. We give a new proof that gives rise to a partition of singular line graphs of trees into two disjoint classes and raises some new questions.

A useful result which enables the recovery of most of the terms of the characteristic polynomial of the parent graph G from the $\mathcal{PD}$ is the following:

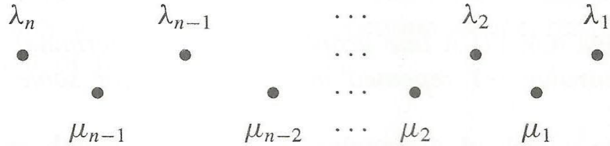
LEMMA 4.1.

$$\phi'(G,\lambda) = \sum_{\mathcal{PD}} \phi(G-v,\lambda) \tag{4}$$ □

Thus by integrating (4), we obtain $\phi(G)$, save for the constant term. Thus a boundary condition is required to determine $\phi(G)$ completely.

LEMMA 4.2. *If an eigenvalue* λ_0 *is known then* $\phi(G,\lambda_0) = 0$. □

Remark 4.1. Let the eigenvalues of G be $\lambda_1, \lambda_2, \ldots, \lambda_n$ and those of $G - v$ for a vertex v of G be $\mu_1, \mu_2, \ldots, \mu_n$. By the Interlacing Theorem the values can be shown as follows:



LEMMA 4.3. *If* $\mu_i = \mu_{i+1}$ *then* $\lambda_{i+1} = \mu_i$ □

This proves the following theorem:

THEOREM 4.1. *If there is a card of* $\mathcal{PD}$ *with a repeated factor then* $\phi(G)$ *is reconstructible.* □

Equivalently if in the deck of spectra, there is a card with a repeated eigenvalue, then $\phi(G)$ is reconstructible.

DEFINITION 4.1. *A singular graph is said to be a nut graph if a kernel eigenvector of the graph has no zero entries.*

LEMMA 4.4. *The nullity of a nut graph* G *is one.*

Proof. All eigenvectors of G are multiples of each other, since otherwise a linear combination of two such eigenvectors, which is also an eigenvector, could have a zero entry. Thus the nullity of G is one. □

COROLLARY 4.1. *A nut graph has no pendant edges.*

Proof. Let G be a singular graph of nullity one with a pendant edge xy such that x is the vertex of degree greater than 1. The entry of the kernel eigenvector corresponding to x is zero. Thus G is not a nut graph. □

The following result follows.

THEOREM 4.2. *If* L_T *is a nut graph, then its terminal cliques are* K_r, $r \geq 3$. □

The following theorem is proved in [8].

THEOREM 4.3. *If L_T is singular and a kernel eigenvector has no zero entries then it is a nut graph. Otherwise $\exists v$ s.t. $L_T - v$ has the eigenvalue zero repeated.* □

THEOREM 4.4. [7] *A line graph L_G with two terminal $K_r s$, $r \geq 3$, has the eigenvalue -1 repeated in $Sp(L_G - v)$ for some v.* □

Thus the s-deck of a singular L_T has a card with an eigenvalue repeated.

THEOREM 4.5. *If L_T is singular then $\exists v$ s.t. $L_T - v$ has the eigenvalue zero or -1 repeated.* □

Thus from the p-deck of a singular L_T, we can deduce one of the eigenvalues of L_T. This provides the required boundary condition that determines $\phi(L_T)$.

THEOREM 4.6. *The characteristic polynomial of a singular L_T is reconstructible from $\mathcal{PD}$.* □

Since the unknown constant term in the characteristic polynomial of G is $(-1)^n \text{Det}A$, which is zero for a singular graph, recognition of a singular graph from the p-deck is sufficient for polynomial reconstruction. An algorithm has been developed that recognises singular line graphs of trees.

Algorithm: [9] Given a $\mathcal{PD}$, the algorithm determines

1. whether the $\mathcal{PD}$ is the legitimate p-deck of the line graph of a tree,
2. whether L_T is singular.

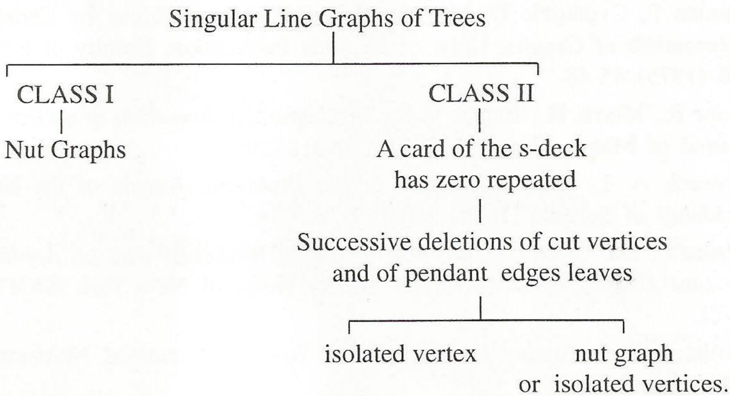
Thus singular L_Ts are recognisable and therefore reconstructible from their $\mathcal{PD}$.

5. Classification.

The discussion on the eigenvalues of singular line graphs of trees, above, shows that this set of graphs can be partitioned into two classes:

i) one class is the L_Ts which are nut graphs and which have a vertex-deleted subgraph with a repeated -1 in its spectrum;

ii) the second is the singular L_Ts which have an eigenvector with a zero entry and thus exhibit a repetition of the zero eigenvalue in one card of the deck.



6. Open problems.

This study was motivated by the desire to characterize the trees whose line graph is singular. We have established that no odd tree has a singular line graph. We were also interested in the possible repeated eigenvalues that appear in the deck of spectra of the vertex-deleted subgraphs of singular line graphs of trees. We have shown that at least one vertex-deleted subgraph has the eigenvalue -1 or 0 repeated. This has proved the polynomial reconstruction conjecture for singular line graphs of trees.

There still remains the problem to **characterize which even trees have a singular line graph.** Moreover, while constructing those line graphs of trees which are expected to be singular, it proved to be very difficult to construct one with a large terminal clique. In view of this we pose the following conjecture:

Conjecture 6.1. If L_T is singular then terminal K_rs are small; i.e. $r \leq 3$. □

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