

## The combination rules of standard uncertainties

$f = f(x, y, z)$	$df = \left(\frac{\partial f}{\partial x}\right)dx + \left(\frac{\partial f}{\partial y}\right)dy + \left(\frac{\partial f}{\partial z}\right)dz$	$u(f)^2 = \left(\frac{\partial f}{\partial x}\right)^2 u(x)^2 + \left(\frac{\partial f}{\partial y}\right)^2 u(y)^2 + \left(\frac{\partial f}{\partial z}\right)^2 u(z)^2$	$\Delta f$
<b>Addition / subtraction</b> $f = x \pm y \pm z$	$df = (\pm 1)dx + (\pm 1)dy + (\pm 1)dz$	$u(f)^2 = (\pm 1)^2 u(x)^2 + (\pm 1)^2 u(y)^2 + (\pm 1)^2 u(z)^2$ $= u(x)^2 + u(y)^2 + u(z)^2$	$u(f) = \sqrt{u(x)^2 + u(y)^2 + u(z)^2}$
<b>Multiplication / division</b> $f = \frac{xy}{z} = xy \cdot z^{-1}$	$df = \left(\frac{y}{z}\right)dx + \left(\frac{x}{z}\right)dy + \left(-\frac{xy}{z^2}\right)dz$	$u(f)^2 = \left(\frac{y}{z}\right)^2 u(x)^2 + \left(\frac{x}{z}\right)^2 u(y)^2 + \left(-\frac{xy}{z^2}\right)^2 u(z)^2$ $= \left(\frac{f}{x}\right)^2 u(x)^2 + \left(\frac{f}{y}\right)^2 u(y)^2 + \left(-\frac{f}{z}\right)^2 u(z)^2$	$u(f) = f \sqrt{\left(\frac{u(x)}{x}\right)^2 + \left(\frac{u(y)}{y}\right)^2 + \left(\frac{u(z)}{z}\right)^2}$
<b>Mean:</b> $f = \frac{\sum_{i=1}^N x_i}{N}$ where $\forall i, \Delta x_i = \Delta x$	$df = \frac{(1)dx_1 + (1)dx_2 + \dots + (1)dx_N}{N}$	$u(f)^2 = \left(\frac{1}{N}\right)^2 u(x_1)^2 + \left(\frac{1}{N}\right)^2 u(x_2)^2 + \dots + \left(\frac{1}{N}\right)^2 u(x_N)^2$ $= \left(\frac{1}{N}\right)^2 u(x)^2 + \left(\frac{1}{N}\right)^2 u(x)^2 + \dots + \left(\frac{1}{N}\right)^2 u(x)^2$ $= N \left(\frac{1}{N}\right)^2 u(x)^2 = \frac{u(x)^2}{N}$	$u(f) = \frac{1}{\sqrt{N}} u(x)$

Note: Recall that when the equipment certificate or other specification gives limits without specifying a level of confidence, or when an estimate is made in the form of a maximum range ( $\pm\alpha$ ) with no knowledge of the shape of the distribution, then we may assume a rectangular distribution function with a semi-range of  $\alpha$  with an associated standard uncertainty (or standard deviation) of:

$$u(x) = \frac{\alpha}{\sqrt{3}}$$