Letters of Euler
on Different Subjects in
Natural Philosophy
addressed to
a German Princess

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paraphrased from http://math.dartmouth.edu/∼euler/

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1 Sizes and Speeds (Letters 1-2)

Let us start our panoramic overview of what is known in Natural Philosophy by setting the stage of discourse — the universe, its range of sizes and motions. Starting from a "foot", we can go down to a ten thousandth of a foot, which is the size of the smallest 'animal' visible. Conversely we can go up in size to a mile, to the size of the Earth, the distance of the Moon, the Sun, and finally the farthest stars at possibly two hundred million billion feet distance.

If we look at motion, we can go from a leisurely walk to running, to a galloping horse, the wind, sound, a cannon projectile[1], the Earth's rotation is faster still, the Earth's speed around the Sun, and finally the speed of light itself.

2 Sound (Letters 3-8)

We are all familiar with sound, but less so of its finite speed. This becomes apparent over longer distances, for example thunder follows lightning by an appreciable time lag (in fact we can use this time lag to calculate how far away the thunderstorm is). Sound is not like smell, it is a wave, not a diffusion of particles. As such, it has pitch (flat/sharp) and amplitude (piano/forte). For example the note C is a sound with 100 beats per second.

Now when a pitch is twice another, we hear a perfect overlap of the beats, and we say they are "consonant", easily discernible even to non-musical ears. So much so that musicians denote both notes by the same letter, but say that one is an *octave* higher than the other. Of course, there are higher consonances, such as double octaves (four times the frequency).

[1] the fastest thing then known on Earth

The ear discerns not only exact multiples of a note pitch, but also simple fractions of its frequency. The note G for example is 3/2 of C's frequency. Of all fractions that involve 2 and 3 in the numerator or denominator, the simplest are in fact G= $\frac{3}{2}$ C which sounds very pleasing when sounded together, F= $\frac{4}{3}$ C, and D= $\frac{9}{8}$ C. The interval CG is called a *fifth*, while CF is called a *fourth*; the smaller interval FG is called a *tone*. We can also add fractions that involve the number 5, the simplest being A= $\frac{5}{3}$ C, E= $\frac{5}{4}$ C, and B= $\frac{15}{8}$ C. The notes ABCDEFG constitute the *diatonic* system. More complex consonances are Fs= $\frac{45}{32}$ C, Cs= $\frac{25}{24}$ C, Ds= $\frac{75}{64}$ C, Gs= $\frac{25}{16}$ C, Bb= $\frac{225}{128}$ C, which constitute the modern musical system, in which the difference from one note to the next, termed a *semitone*, is approximately 16/15.

What we call music is a certain balance in complexity of such notes.

3 Air (Letters 9-16)

The surrounding air certainly exists because we can feel it when we move our hands, and when we feel the wind. It can be compressed for example by a pump, but water cannot. This shows that everything around us can be divided into solids, which do not flow, and fluids which do; and the latter can be further divided into liquids, which are incompressible, and gases which are.

Apart from compression, air can also be rarified. Air has a certain quality, termed *elasticity* [nowadays called pressure] which is the power of air to expand. So given the chance to expand it will. For example, take an air pump attached to a vessel, pull its piston out, close the vessel, push the piston back in, and repeat the process. The air in the vessel keeps on expanding until what is left inside is close to a vacuum. Such an empty vessel weighs less than one full of air, hence air has weight. Reflecting for a moment, how much air is in the atmosphere, it is clear that air here at the surface of the Earth has a higher pressure than high up in the atmosphere.

This pressure can in fact support an amount of water, and this is the principle behind a barometer. To show that it is the air which is supporting the water, place the barometer inside a flask, and remove its air – the water will run out. This air pressure, when released suddenly, can be impressive, as for example happens in an air gun, or even more dramatically in a gun, when gunpowder suddenly changes into 'air' once ignited.

Another property of materials, including air, is temperature. Materials lengthen when heated – a pyrometer measures temperature by amplifying this increase using a lever attached to a metal bar. Similarly a liquid thermometer works by forcing the expanded liquid up a narrow tube. It is a law for air that its pressure is proportional to its temperature and its density. Since temperature must vary from place to place on the globe, there must certainly be differences in pressure, which is the cause of the wind that blows fairly continually throughout the year.

One curious paradox is the observation that it is cold on mountain tops even in summer and even at the equator. Why this is so may appear perplexing, but note that transparent objects allow light (and its heat) to pass through them, so that it is not heated (unless the air is foggy). So practically all the sun's heat is received at the surface of the Earth. Mountain tops, tiny in comparison to the surrounding cold atmosphere, lose all their heat easily. But it is still cold in Quito, which lies at the equator and is not on a mountain top – why shouldn't it be like Paris? The answer seems to be that air is less dense and humid the higher you go, and this makes it colder. Conversely in mines, where the pressure and humidity are high, it is always warm.

4 Light (Letters 17-44)

There are two schools of thought concerning light. Descartes thought they were waves in ether (albeit with infinite speed). Newton thought they are rays of substance passing through vacuum. But the latter seems to suffer from various difficulties — how come the sun does not seem to diminish in size after losing all that substance continually? when two light beams pass through each other, nothing happens, but shouldn't colliding particles affect each other? how can objects be truly transparent? and how can a small candle emit particles at such a high speed? Descartes' ether has its own problems - why don't planets encounter resistance? This problem probably forced Newton to decide that planets reside in vacuum, through which no wave can pass.

But Newton's solution is not consistent, because he immediately proposes that light is a substance, so that space would be filled with this substance anyway, and would still perturb the planets' motion.

The only compatible solution seems to be to assume an ether so rare that planets do not encounter resistance, yet at a very high pressure. Light is then a wave in this ether, just as sound is a wave in air. That light has a speed that is 900 000 times that of sound is easily explained because the wave speed is proportional to the pressure divided by the density of the medium. Ether has a much higher pressure and a lower density than air and so explains the difference. Even so, it still takes about 6 years for light to travel from the nearest star, let alone from stars that are very faint and hence farther away.

The Sun must be very hot to produce all the light and heat waves. Can this possibly go on forever? But the sun is very large so it would take a long time to 'consume'.

Light must have a very high frequency, because sound does not produce light. Objects either emit light of their own accord, termed 'luminous' (for example, the sun), or they reflect light (for example, the moon). Ultimately most of the light comes from the sun (and stars). Note that light is independent of us, unlike what the ancients claimed — simply shut all windows in a room, and you can't see a thing!

Reflection of light is a familiar phenomenon. According to Newtonians, it occurs in a similar fashion to a billiard ball collision. The Cartesians though liken in to a sound echo. Note that there is a difference between reflection off mirrors and reflection off opaque bodies, something that is often neglected by philosophers. The first is true reflection, whereas the second is a secondary reflection and can change characteristics of the light such as colour etc. This means that either the light particles or waves are being changed, something that is not allowed in true reflection. The main difficulty to be explained is how objects appear in a single colour irrespective of the light striking it. The probable answer is *resonance*. Recall how one string in a musical instrument vibrates of its own accord when placed near to another, in harmony. Or how a powerful voice can even shatter a glass through resonance. Thus of all the light hitting an object's surface, only a few frequencies resonate with the object's particles, which then *emit* light of that frequency.

The colour of light corresponds to its frequency, as pitch is to sound. Newtonians however believe that colour is an intrinsic property of the light particles, and objects appear colourful because only certain particles are reflected, the others absorbed. Still, both theories agree that white is a mixture of all colours; white objects appear so because they contain particles that resonate at all frequencies, and black objects contain particles that resonate at no frequency.

Transparent objects are never truly transparent, just less opaque. Thick transparent material is opaque (glass), while thin opaque material (gold leaf) is transparent. This is similar to sound passing through thin walls or doors.

Refraction is the bending of light when passing from one transparent medium to another. It depends on the density of the media and on the light's colour. Thus a rainbow results through refraction and reflection of light through myriad droplets.

Why is the sky blue? Simply because it contains tiny blue particles sparsely interspersed in the atmosphere. It becomes visibly blue over large extents of air, not only vertically upwards but also when one looks at far away mountains.

Light emitted from a source becomes fainter the further one moves away from it, in fact precisely as an inverse square relation. Thus a star appears very faint, not visible in daylight, even though the source is as bright as the sun. In like fashion they appear smaller. Of course the apparent size of an object is not its true size. Although the moon and the sun both appear the same size, they are vastly different in true size. When we perceive the world around us, we can only discern colour, shape, apparent size and direction. Distances (and so true sizes) are only inferred by our brain. Yet this does not mean, like some empiricists and skeptics claim, that the senses deceive us.

Which brings us to another curious paradox: why does the full moon (and the sun) appear larger on the horizon than when high up in the sky? There is no clear-cut answer, but probably we infer their large size because they appear slightly fainter at the horizon. An alternative is because we compare them to objects, and infer that they must be very far away.

Shadows of objects come in three types, depending on the relative size of the luminous source compared with the opaque object. A small source leaves a shadow increasing in size behind the object; equal sizes gives a cylindrical shadow with partial shadows surrounding it; large sources leave a finite conical shadow, with partial shadows. Eclipses are then understood as the motion of the moon through the Earth's shadow, or vice versa. Twilight is the result of the air at the top of the atmosphere moving out of the Earth's shadow a few minutes before the place beneath it.

Mirrors are worth a study in themselves. The basic rule is that the angle of reflection equals that of incidence. A little geometry convinces one that the apparent object lies at the symmetric point on the other side of a plane mirror. Convex mirrors, such as a spoon exterior, produce small images, while concave mirrors, such as a spoon interior, enlarge them. Moreover they can focus light to a point, causing it to heat up, even to melt some metals, and for this reason are called 'burning mirrors'.

Lenses also come in two types, convex ('burning glasses') and concave. The camera obscura makes use of the former to produce sharp images at the back.

Eyes are a 'perfect' combination of lenses that focus images at the back of the eye, called the retina. The pupil/iris is there to control the amount of light that enters. When outside, it is small, and remains small for a while when one enters a dark room, which thus appears darker. Conversely, once inside the pupil is enlarged, and remains large for a while when one goes outside, giving a dazzling experience. Note that although lenses are chromatic, the eye is not. This must imply that there must be a combination of achromatic lenses.

5 Gravity (Letters 45-68)

That things fall until they meet an obstacle, hardly needs mention. The ancients thought there were exceptions to this, such as smoke and vapours, but we now know that they rise because the air falls more forcefully. Smoke on its own, inside a vacuum also falls down. More generally, objects float only on heavier (denser) fluids; even 'heavy' lead will float on mercury.

Furthermore, all objects fall with the same acceleration. Even things that are thrown up, must eventually fall. It might be possible that some materials have no weight, perhaps light, electricity, magnetism. The downward weight of an object is just one force among many others, such as air pressure and resistance. Weight is additive, but depends on the type of material.

This preference for a downward motion is the source of our words "vertical", "up", "down", "horizontal". Water flows ever downwards when possible, witness rivers. A spirit level is perfectly horizontal as it has nowhere to go.

We all know that the Earth is spherical, so we should qualify the direction 'down' to mean towards the centre of the Earth. Imagining that at an antipode people walk on their heads is mere nonsense. Because of the Earth's spin, objects weigh slightly less at the equator. The weight of an object must vanish as we go high up, but it is still significant even at the Moon's distance. How come the moon does not fall down to Earth, then? The answer is that the Moon is not at rest. A cannon ball fired at higher and higher speeds will fall farther and farther away, eventually it would reach the antipode, and at higher speeds the ball would become a small moon. So the Moon is actually falling, but moving horizontally sufficiently fast that it perpetually keeps its height above the Earth.

If an apple falls to Earth by a force, then the Moon, also made up of matter, must also be falling in the same manner. Newton was thus led to his universal law of gravitation. Any two objects, whether Jupiter and its satellites, or Saturn and its satellites, or the Sun and its planets attract each other. This force acts at a distance, similar to the magnetism of a lodestone. Moreover it is reciprocal, that is the Moon attracts the Earth with the same magnitude. This is confirmed by observations about how planets' and comets' orbits are perturbed by other planets. Stars though, are so far away, that they cannot affect us appreciably.

But, one wonders, why don't objects on a table approach each other if there is this mutual attraction? The reason is that this force is extremely weak for small objects. It does become appreciable with objects the size of a mountain, as some experiments in Peru have shown.

The exact law of the gravitational force is that it is proportional to both the objects' masses, and inversely proportional to their distance squared. Hence the sun has a major gravitational effect on every object in the solar system. Thus planets move primarily according to the Sun's pull, the other planets' attraction being much weaker due to their distance. For the Earth in particular, the greatest forces are due to the Sun, and secondarily to Venus when it is close to Earth, and less so to Jupiter and the Moon. For the Moon, it feels both the Earth's and the Sun's pull at comparable magnitudes, leading to a complicated motion.

It is quite probable that all the other planets and moons are inhabited, as they appear very similar to the Earth's landscape. Stars also probably have their own planets, making us wonder at the universe's grandeur. Leibniz has even dared suggest that the universe is as best as possible, to which Voltaire's sarcastic suggestion was that evil is also the best possible.

Tides are explained by this theory of gravity. Tides have a natural period of 12 hours in pace with the moon's position; more so at full or new moons, and at spring/autumn equinoxes. They are larger in the oceans than the seas, especially at the equator. Now, every object on the Earth's surface, such as the water in the sea, feels primarily the Earth's pull, with the Moon and Sun having a much weaker effect. So what causes tides, and what are their relation to the moon? Descartes had attempted to explain it via the planetary vortices' pressure. But the true explanation is that the average force on the water is the gravitational pull due to the Earth together with the average pull from the Moon. But the Earth is ever spinning around its axis, so that at any place, the Moon's pull would rise from an average to a maximum then decrease to a minimum every day. When at a maximum, it partially cancels the Earth's pull, and the sea level tends to rise (with a time lag though); when the Moon is on the other side of the Earth, the net force on the water is less than the average, and so the water rises again from its average level. This explains why tides have a period of half a day. The same tidal forces (but about a third of the strength) occur for the sun's gravitational pull. When the solar and lunar tides reinforce each other (at new or full moon, especially at the equinoxes) the tides are larger.

What causes gravitation? For Newtonians, the gravitational attraction is fundamental in itself, not requiring an explanation. For Cartesians, action at a distance is absurd, and it is the ether that is the ultimate cause of the force.

6 Mechanics (Letters 69-78)

A body is defined as a volume which can be moved, and most importantly, has impenetrable matter. Bodies are only at rest relative to something else, for example we are all moving along with the Earth. The basic laws of motion of bodies are the following. Firstly, bodies cannot move themselves, and so must remain at rest or in uniform motion if no force acts on them. Note that Aristotelians thought that things tend to stop moving of their own accord; after all everything stops unless forced to move. But this observation still fits in well with the first law, when one considers friction and resistance. This property is called inertia, and is proportional to the amount of matter in the body.

A force (also called power) changes its velocity. For example, the gravitational force pulls objects downwards. Thought: If any two particles attract and forces produce motion, then everything must be continually changing forever. This need not be the case because forces may cancel out.

The ultimate origin of forces is the impenetrability of objects. The renowned principle of least action can be understood as the following — among all possible changes, the force that causes them is the least possible.

7 Soul (Letters 79-101)

Gravitation is not the only force. The soul acting on the body (causing it to move) is an example of a spiritual force. Materialists like Descartes deny this, and consider animals to be machines, but this is hardly worth debating. The soul receives sensory data and controls muscles. Leibniz denies this force as well, but posits a 'harmony' between the soul and the actual motion.

The essential property of the soul is its liberty. Some argue that men do not choose freely but according to motives. But even the uneducated know this to be false — tell a general to excuse a soldier for deserting against his will, and you'll soon realize this. Miracles are sure possibilities, when God wills (forces) matter to move.

One must distinguish between natural events such as tempests, whirlwinds, eclipses and so on, and man-made events such as wars. Earth was created as the best possible — look at an eye, at living things in general. But what about evil? Is it created wilfully or is it necessary? We cannot fathom the answer but one must admit that good does come out of evil.

The body and soul communicate through the 'corpus callosum'.

Idealists assert that things do not really exist. After all, dreams appear real until we awaken, so how do we know this 'reality' is not in fact a dream, from which we will be awakened one day? As a more practical example, we can never be sure that an object has remained the same if we stop observing it.

The soul has various aspects of thought: perception (from senses), imagination (from memory) and abstraction, in which we focus on a quality of objects, not the object itself. This abstraction can be taken to various degrees and we get the classification genus-species-particular. Finally, language enables us to grasp these thoughts and communicate them.

8 Logic and Knowledge (Letters 102-Volume II 17)

There are four basic statements (i) every A is B, (ii) no A is B, (iii) some A is B, and (iv) some A is not B. Statements (i) and (ii) are called universals, the other two are called particular, (i) and (iii) are called affirmative, the others negative.

The knowledge of two such statements can allow us to conclude a third one. But this is not true for any two statements. The syllogisms are the rules for correct inferences. For example if every A is B, and every B is C, then every A is C. There are nineteen other types of syllogisms.

Note that when the two statements and the conclusion are all true, it may still be the case that the conclusion does not follow from the statements. For example, some learned men are misers and no miser is virtuous. From these, one cannot logically conclude that some virtuous men are not learned, even though this statement may in fact be true. For had it been the case, then we would by right conclude from "some trees are oaks", and "no oak is a fir", that "some firs are not trees."

We notice that from two negative statements, or two particular statements, nothing can be concluded. A negative and an affirmative statement lead to a negative; a particular and a universal lead to a particular, while two affirmatives lead to an affirmative. Also note carefully that a particular instance (say Virgil is a poet for example) should be treated as a universal.

Even more general than syllogisms, are the inferences: if A then B is true, and A is true, then B is true; if A then B is true and B is false, then A is false. Note the fallacies, if A then B is true, and B is true, then A need not be true; if A then B is true and A is false, it need not follow that B is false. For example, the newspapers are saying that peace is around the corner, and then peace is achieved, are we to conclude that the newspapers speak the truth?

Knowledge consists of three types of facts:

(i) Sensory facts, obtained by first-hand perception, "I saw it"; experiments fall in this category.

(ii) Inferred facts, obtained by using logic; geometry, and science fall in this category; truths are argued not sensed.

(iii) Authoritative facts, obtained by reference to someone credible; history and religion fall in this category. These facts are themselves divided into three, according as the authority knows the fact as a sensory, inferred or upon other authorities.

Deception occurs in all three categories. Senses may deceive us, we can argue from wrong premises or using faulty logic, and rumours are certainly not all facts. Yet we cannot conclude that all facts are deceptive in nature. One just has to exert care when accepting a fact.

9 Colour and Sound revisited (Letters 18-22)

Descartes thought that colours were shades of white/black. But surely coloured objects retain their hue in both sun and shade.

Sound is characterized by amplitude, frequency and 'timbre'. Two notes may be of the same amplitude and frequency, but a flute and a horn certainly sound differently. By altering these characteristics it is possible to emit any sound. It should even be possible one day to create a machine that emits speech.

10 Electricity (Letters 23-39)

Electricity has long been known of, but recently a spate of experiments are bringing more knowledge of this difficult subject. Electricity is generated by friction on glass or sealing wax with wool. The charged glass will then attract and repel paper. Bringing two charged glasses can even create sparks between them.

There are two types of bodies in relation to electricity. Either an object can be charged by friction, or it can only be charged by touching with a charged glass, such as metals and water. Electricity does not pass through the body in the first case, and so is called an insulator, while it does in the second case, and is called a conductor. Wire conductors can convey electricity long distances. It is thought that lightning is electricity on a grand scale.

[In what follows, Euler goes to great lengths to explain how electricity is nothing else but the flow of ether. Conductors have large pores through which the ether flows easily, while insulators have tiny pores which can trap the ether at high pressure. These explanations will be omitted here.]

Consider the following experiment. Charge a rod of glass and place a finger near it. A spark will issue between the finger and the glass (the air must be dry). For better effects, take a rotating tube of glass and apply a dry cloth to it. The glass becomes much more charged than before.

To such a tube of glass, attach a wire (or metal chain) to a piece of iron suspended in the air by insulators. As the tube rotates, the iron accumulates charge. Placing a finger near it creates a big spark. Touching the iron (while insulated) causes the body itself to be charged. In the dark, an eerie light is seen around the body. Some have assumed this is healthy!

Now there is a difference between a glass charged this way, and charged sealing-wax. Touching two glass spheres charged in the same way, does not produce sparks; touching sealing wax to glass produces a strong spark. We arbitrarily denote the glass to be positively charged, the sealing-wax to be negatively charged.

Leyden has performed an interesting experiment, in which he transferred the glass charge to an iron bar, and the cushion charge to water inside a glass flask placed inside a water tub, with a metal chain connecting this water to the earth. This arrangement produces very strong shocks, certainly unhealthy.

11 Determining the Longitude (Letters 40-57)

To determine one's position on the Earth is quite probably the foremost scientific problem of the age. Positions are calculated using latitude and longitude angles. Different countries measure the longitude with respect to different reference points, but this is immaterial.

To determine latitude is a straightforward measurement of the elevation of the polar star.

To determine the longitude is a much more difficult problem. Normal dead-reckoning (i.e. plotting a course on a map using the estimated distance travelled each day) should work but errors, apart from tempests and sea currents, make it a very unreliable method.

There is another way, one that involves a precise clock. Look at the time on this chronometer when the Sun is at mid-day, and the difference from 12 noon, multiplied by 15 degrees per hour, gives the longitude. The problem is that clocks that remain precise in the rigours of travel are not available. The most accurate clocks are pendulums, but these are obviously useless at sea. There are some claims to accurate clocks but nothing has been heard of them since.

So one must look at the skies to furnish us with natural precise clocks. The Moon's eclipses are obvious ones. One can easily create accurate tables of times when such events should occur. Then measuring the local time of the eclipse (from mid-day using a common watch) and comparing with the almanac time would give the longitude as above. The rarity of eclipses make such a method useless except for determining the longitude of a fixed place. However the Moon's position can be used for this purpose as an alternative. As a result of improvements suggested by myself, it is now possible to have almanacs of the moon's position accurate to within 1' of arc, which translates to an acceptable $1°$ longitude error. Similarly one can use time-tables of Jupiter's satellites' occultations, except that one would need a steady telescope to view them.

A further speculative method involves the magnetic compass. A magnetic needle points north plus or minus an angle called the "declination". This declination varies from place to place, and with time. Halley has plotted a chart of declinations on his voyages, and found that places with the same declinations follow lines that look like longitudes except more complicated. Nevertheless, finding a declination and latitude would give the exact position if we had an accurate world map of declinations for that year.

12 Magnetism (Letters 58-71)

Natural rocks, called loadstones, obtained from mines, have the property of changing direction when hung by a piece of thread or when placed on a piece of wood floating on water. They always point to "north and south". Two loadstones turn to face each other, with opposite poles adjacent. Loadstones with the same pole adjacent repel each other.

Some 'naturalists' explain the earth's magnetism by putting a powerful magnet inside a globe. They observe very similar results of declination as are actually observed. Although a question arises - if the declinations change yearly, then the Earth's loadstone must be changing its position or direction, how is this possible?

Iron filings placed on a card on top of a magnet trace out a nice pattern, suggesting a magnetic fluid going round in a cycle. This fluid would only affect iron (and loadstone) and is not affected by the presence of air or not. This fluid motion would explain the attraction and repulsion of the poles.

A magnet can be made by stroking or by hammering in a vertical position. Given a magnet, a piece of iron placed *near* it itself becomes a magnet. A loadstone can be made more powerful still by putting it in a soft iron armour. This increases the magnetism of its own accord.

13 Optics (Letters 72-110)

France has a burning glass 3 feet thick. It can burn wood instantly and even melt metals. To get an idea of the temperatures involved, we have a clear idea of the temperature of our bodies and the outside; water boils at about three times this temperature; lead melts at three times this (9 times body temp.), and copper at three times this, with gold a little bit higher. This is the power of concentrating the sun's rays to a small focus.

We have all seen a camera obscura in which a lens projects a small image at its back. A magic lantern works in the same way in reverse. We place things or painted glass in a small box with a good light, and project the image via a lens to a wall.

A simple microscope is a powerful lens. But there is a limit how powerful such lenses can be. A magnification of 30 is easy to achieve, but for more one needs a shorter focus which in turn requires a small lens. Tiny lenses with magnifications of 200 are known, but the image appears very faint.

To achieve more, one needs to compound lenses together, which brings us to the principle of a telescope. The first telescopes consisted of a convex lens (called the 'objective' because it is aimed at the object) and a concave lens (called the 'eye piece'), placed so that their foci coincide. Magnifications of 10 are easily achieved this way. But this type of telescope has a big disadvantage in that the view is very restricted; more powerful lenses would make this worse. A bigger field can be obtained by making the eye-piece also a convex lens, and this allows for magnifications of up to 100. Its disadvantages are that the image is inverted, and is quite chromatic. Its use is therefore restricted to astronomy, and even then, reflecting telescopes are used to avoid chromaticity. However even with such powerful telescopes, we cannot hope to see the people and animals that inhabit the Moon, that would require magnifications of more than 200 000.

Two defects that merit special attention are the difficulty of grinding a lens that retains a fixed focal length at various angles of incidence. I exerted an immense effort to find a combination of radii for the lens to remove this aberration. I also managed to find a combination of lenses of different densities (water and glass) that is also achromatic, although with less success.

For terrestrial telescopes, a combination of four convex lenses are used, the first two play the same role as before, and the last two re-magnify and re-invert the resulting image.

What do we see with these telescopes? That the moon has mountains, that planets are discs, that stars still appear as points, and there are many many more stars than are visible with the naked eye. This must mean that they are very far away, at least 20000 as the distance from the Sun to the Earth.