Galileo Galilei

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last revised 26 Nov 2008

A paraphrased summary of the 437-page book in zope.mpiwg-berlin.mpg.de/-archimedes, and the 313-page translation in oll.libertyfund.org

1 Dialogue on Two World Systems (1632)

1.1 The First Day

The two greatest World sytems are due to Ptolemy and Copernicus, following Aristotle and Aristarchus (and the Pythagoreans). This book is divided in three parts: (i) I show that it is impossible to perform experiments that show whether the Earth moves or not, (ii) I examine the evidence for the Copernican theory, (iii) I propose to explain the tides using Earth's motion. It is written as a dialogue between two old friends of mine, Sagredo and Salviati, and Simplicio[1], after a Peripatetic champion of the Aristotelian faith, who shall remain unnamed.

Salv.: Aristotle states that 3 is the perfect number since it is the number of All, beginning, middle and end; yet isn't 2 better as regards feet, or 4 as regards elements? This is just mystic numerology.

Sagr.: From this he deduces that the dimensions of a body are 3. Yet one can prove this simply by observing that one can draw only three perpendicular lines from a point, and no more.

His assertion that there are three simple motions is also suspect: there is also helical motion, which is like itself everywhere. Again, why did he take upward and downward motion along a straight line as the only simple natural ones, when in fact there are 3 independent directions he could have taken? If these are taken with respect to the Center, then why does he take only one circle? Neither does he allow objects to move with a mixed motion, but allows only a simple motion of the dominant element. It seems that Aristotle is shaping his principles to suit his views.

He has us believe that there are three simple bodies corresponding to the simple motions; how are we to know which is the simple element, Lead or Earth or Wood, since all fall the same way? His whole structure is based on the distinction between a heavenly quintessence moving in a circle, and worldly elements moving in lines.

Salv.: Let me show that when an object falls, it accelerates in velocity from zero to the final one, going through all lesser velocities in the meantime. I will make two assumptions:

(i) that the downward impetus, if reversed upwards, would take it to the same height (as a pendulum while swinging down, goes up to almost the same height, or water going down a pipe reemerges at the same height); the impetus does depends on the height fallen, so that a ball falling down an inclined plane

[1] Galileo's patron, Galileo, and Pope Urban VIII? Simplicius, a derogatory choice of name, was a 6th century philosopher

has an end velocity the same as if it had fallen directly down, for it is not any closer to Earth's center.

(ii) if we take the limit as the inclined plane is made ever more horizontal, the time it takes to fall down it will be an hour, a year, as large as we want. It follows that the speed down the incline is as small as we want; but this speed is the same as that of a ball falling directly the same height.

Simp.: Yet the time it takes for a direct fall is less than for an inclined one. How can it be that they go through the same velocities and take different times? A velocity means that it takes a certain time to achieve a certain distance. So, something falling directly down has a greater velocity than one falling inclined.

Salv.: It will be agreed that a ball will take less time to fall down a more inclined slope, and the perpendicular the least. Elsewhere I show that the velocity at the end of the inclined plane is twice the ratio of the length of the plane to the time taken to fall.

In the limit that the plane is horizontal, there will be no motion at all. As this is what happens if it were to move in a circle, it follows that circular motion cannot be acquired naturally; only after given a certain velocity will it be able to start its circular motion. For example, if we suppose that the planets orbit the Sun, and that they started from some point, accelerate towards the Sun, then at some point, when they have acquired a certain velocity, they are turned round to follow a circular orbit. If we know their distance from the Sun, their velocity would be known, and one can retrace the point where they would have started; will we find that they were all created at one place? I have done this long and painful calculation, and found a good correspondence with observation[2].

There are two motions that conserve order: at rest, or in uniform circular motion. Motion in a straight line is necessarily accelerating, and so cannot continue so.

One cannot show from this that the Earth is at the center of the universe. For it may be argued that earth and water move towards the center of the Earth instead. If every part goes to its whole, it would explain why not only the Earth, but all the heavenly objects, are spherical.

Simp.: Aristotle, the master of logic, demonstrated that Earth is at the center, and that the heavenly bodies are not made of matter, neither heavy nor light, nor destructible into parts.

Salv.: Ah, but an organ-maker need not be an organist!

Sagr.: One might perhaps argue as follows: fire goes vertically up away from the World's center, and earth down towards it; since we see the same happening all over the spherical world, the center of the World must be the center of the Earth.

Salv.: All that this shows is that matter move towards or away from the center of the Earth. If we remove the distinction between heavenly circular and earthly linear motions, we lose Aristotle's elevation of the quintessence as a divine substance, impenetrable, without weight etc. Rather we would be placing Earth in heaven.

[2] As asserted in the second Dialogues book

Simp.: To say that the Heaven is the same as the Earth! Isn't it clear, as Aristotle says, that the heavens have never changed, and rotates uniformly, in contrast to Earth's ageing, irregular, and corruptible things? If you take a false position, you can argue anything.

Salv.: The argument that the Earth cannot move in a circle because it is corruptible hinges on the topic of change. If you know so much about this, illuminate me why it is that some are more corruptible than others: why horses live longer than stags, olives than peaches.

Sagr.: If you accept that corruptible follows by definition from contrariety (as Aristotle did), then the Heavens are corruptible since the sky is transparent, but the stars not.

Simp.: But density in transparency is not the same as density in weight, which follows from the principles of hot and cold.

Sagr.: In passing, you will find that a red-hot iron weighs as much as a cold one. But even so, how do you know that the celestial substance does not differ in the quality of heat?

Salv.: I am sure that Aristotle and Simplicius would not accept that there will be a time when the Earth exists no more; that corruption is therefore of its parts, not of the whole; but then, using the same logic, the Earth as a whole, ought to move in a circle around the center of the World, or stay at rest.

But we are a-sailing without compass and rudder, going where the wind takes us. Let us forego these bottomless arguments, and instead examine the direct evidence, observations and experiments, for and against the Aristotelian view.

Simp.: For a start, the heavens have never been observed to alter in the minutest detail, in contrast to Earth's decaying life, storms, etc. Secondly, the earth is naturally dark, the sky fully lit up with stars, the Moon and the Sun.

Salv.: As to the first, then America and China are also heavenly, because we never see any alteration on them at this distance. From not *seeing* a change, it does not follow that there is no change.

Simp.: Ah, but there have been changes on Earth that could have been visible on the Moon. There are ancient records stating that the Gibraltar straits once separated, admitting in the sea to form the Mediterranean.

Salv.: Has anyone drawn the face of the Moon so carefully that we can compare and find any changes over the ages?

Sagr.: And to state that one has never seen a star being born or dying is totally unfair: who has seen a terrestrial globe being born or dying?

Salv.: Even so, in our own age, two new stars, *novas*, have been observed by the renowned astronomers Tycho and Kepler, one in 1572 and another in 1604. Apart from the comets, which have been observed to be beyond the Moon. And what look like clouds as big as Africa and Asia combined, have been observed on the Sun. This has been made possible thanks to the telescope, which has brought the skies thirty or forty times closer.

Simp.: The comets have been shown to be of the Earth by anti-Tycho[3], who

[3] Scipione Chiaramonti

also doubts whether the novas are really true. The sun-spots may be an optical illusion of the telescope, or an atmospheric effect; more likely they are objects in orbit that happen to be seen when they are aligned together[4]. In fact they are observed periodically.

Salv.: Observation goes against this explanation: the spots are sometimes seen to appear in the middle of the Sun; and they are clearly seen to be on the *surface* of the Sun by the way they seem to foreshorten and slow down at the Sun's edge. It is not true that they are periodical, except that some spots have been known to go round the Sun in less than a month and reappear on the other side. Finally anti-Tycho's analysis of the novas is reprehensible: he selects and mixes 12 observations, none of which agree to within thousands of miles; is that a refutation? And if one considers how sensitive is the computed distance to small errors in measuring the angles, and the effect of refraction, this is not surprising. Rather, considering that it kept the same relative distance to the surrounding stars, we can be confident that it resides among them.

Sagr.: Simplicio is hesitant, because he knows what this means: if Aristotle is deposed, what will happen to all those teachings based on him, that certainty found in his books, those lectures taught at university, that grand palace built over the centuries by hundreds of intellectuals? Rather he would prop it up than let it go to ruin.

I rather delight in the alterations and changes that occur on Earth; only a dead desert is immutable. If jewels and gold are precious, so much more would be base soil, or a seed, if they were as rare.

Simp.: It befits Earth to be alterable, just as it is fitting that the celestial bodies be immutable, needing nothing more than motion and light to service Earth.

Sagr.: If the Earth as a whole is unchanging, yet alterable in its parts, why is it so much to let the heavenly bodies be the same?

Simp.: Because the changes that would occur on the Moon, say, would be in vain, and Nature does nothing in vain. Everything has a purpose, and that purpose is Man. What purpose would a changing Moon have, unless you are saying that there are men living there.

Sagr.: To speculate what might go on on the Moon is like a forest-man trying to imagine a sea-world. Even when we try to imagine other beings – sphinxes, chimeras, sirens, centaurs – they are just mixtures of what we see.

Salv.: I have seen the Moon through the telescope – it is spherical, reflects the Sun's light, and seems to consist of earth in the form of mountains and crags. There are circular 'banks' with what look like mountains at the center; the smaller ones are very numerous. There also appear to be dark seas.

If there are men on the Moon, the Earth would appear to change monthly, with crescents etc. Yet they would see a daily rotation of the Earth; we don't see the same of the Moon because its rotation matches its motion about the Earth. Actually we do see a bit more than half the Moon, as confirmed by the telescope, for the line joining a person to the center of the Moon is not identical

[4] Fr Scheiner's hypothesis

to the line joining the centers of the Earth and Moon. The Earth would shine brightly in the lunar night.

If there were life on the Moon, it would be quite different from that on land. For the Sun shines uninterrupted a whole fortnight; imagine what torrent heat there is. Also, I have never observed a single cloud or river, but pure placid serenity.

Simp.: The Moon is spherical, but polished like a mirror to reflect the Sun's rays. What look like mountains are variations in the opacity of its surface. Isn't it ironic that what you call seas are dark, when they should lucidly reflect the Sun's rays even more than the land? To imagine how these dark cragged rocks around us could reflect light in the manner that the Moon does!

Salv.: Ah! Come now, look at that sunny wall with the glass window opposite us – see how brighter is the wall than the glass! And before you start to protest about the reflection off the glass, let me remind you that the Moon is not as bright as the Sun, nor does the reflection come from one point, as it would in a polished sphere.

Simp.: Why assume it to be equally polished everywhere?

Salv.: Even so, the central region of the Moon, receiving the rays perpendicularly, would be brighter, and we don't see that. This shows that the surface of the Moon is not smooth but mountainous. In any case why should an incorruptible Moon be so perfectly spherical, would a spherical wooden ball survive longer than an oblong one? Isn't a Moon made of normal cragged rocks, a more reasonable explanation than a highly polished one with just the right reflectivity and opaqueness to make it appear the way it does?

Sagr.: As to your incredulity that the Earth could shine as bright as the Moon, listen to this: the Moon appears very bright in comparison to the night's darkness; it is much fainter during the day. To make a fair comparison, we would need to see the sunny Earth at midnight, so to speak, but that alas is not in our power. One can only imagine how dazzling the clouds and snow would be! We perceive the Moon to be bright, but it is less than the daylight inside this room, reflected from off that wall; try to read a book at night to compare.

Simp.: But look at the dark side of the Moon, especially at the outer rim in twilight, there is some light still, and this comes from the fact that the Moon is very slightly translucent, allowing some sunlight through its body.

Salv.: As to your other quip about lunar seas, let us pour some water on this brick, and tell me if it doesn't become darker. But as I cannot assert positively what substances the Moon is made of, this is idle talk.

We cannot hope to know everything: a grape-planter may know a little about where and when to plant, but how insignificant is that to the Divine knowledge who guides its roots to nourishment and growth of each of its tendrils and leaves! If Michelangelo achieved much by copying a posture of man, how much bigger is the Artist who designed even the lowest worm!

Simp.: You are now praising Man and his intellect, moments after giving up on knowing anything.

Salv.: True, we don't know much, but what we know we know surely, and by that I mean the mathematical sciences, of which we can be as certain as God himself.

Sagr.: Man's intellect is truly in His image; his art, music, discoveries, but most of all his writing, that he can communicate with someone in India, nay, with someone a thousand years from now.

1.2 Second Day

Sagr.: Yesterday we talked about quintessence and earth, the Moon and the Earth, and it strikes me that Aristotle is held in such high authority when the facts are not clearly cut.

Simp.: Aristotle's authority comes solely from his powerful arguments, and his abundant writing about practically everything. Why do you think he still has a reputation after more than a thousand years?

Sagr.: Ah, there is a much shorter book which contains all the truths: the alphabet! You can make it say anything by rearranging its contents! Just as statues are found in stones, or predictions in horoscopes, or alchemists' interpretations in the ancient writings. If Aristotle were present now, and sees the new discoveries, he would be the first to bury his books. It is his followers that put him on a pedestal for worship, now too timid to admit that he was mistaken.

Simp.: If not Aristotle, then who to guide us?

Salv.: We need guides in unknown forests, but in open plains only the blind need them. If you prefer historical texts and quotes than nature itself, then call yourself historians not philosophers. But let us start today's dialogue, about Earth's motion or lack of.

There is a general daily rotation of the sky and all its bodies: Aristotle and Ptolemy attribute it to the sky, while Copernicus attributes it to the Earth; but logically, they give the same appearances. Both agree that any movement of the celestial bodies apart from this is due to the body itself.

Sagr.: Actually, Copernicus ascribed another annual motion of the Earth, which would have unobserved stellar consequences.

Salv.: True, but let us take it step by step. One cannot distinguish the two systems on this point, because motion is relative. But as we admit that the sky is millions of times larger than the Earth, their daily motion would require an unreasonably large speed. It is like going up a steeple and demanding that the city turn round to avoid the trouble of turning your head. Isn't it much simpler to make one rotate and let the innumerable bodies of Heaven rest? Moreover, the planets need two opposing motions in one system, but a single one in the other.

Sagr.: Aristotle conveniently disallows contrary circular motions, saying they are the same. But tell me, Simplicio, if two knights tilt against each other in the field, is that contrary motion? Naturally yes; but in fact the Earth is circular, so they are actually moving in opposing circular motions.

Salv.: In any case, there is order in the second motion of the planets, with Saturn slower than Jupiter, slower than Mars; this is very similar to the way the

Medicean stars[5] move around Jupiter, the closest being the fastest. Whereas their daily motion is all identical.

And when we come to the stars, some move at inconceivable speeds, others near the pole move in tiny circles. With a rotating Earth however, the stars continue the trend of the planets, being at huge distance and motionless. Not only this, but it was found that certain stars that 2000 years ago were moving in circles, are now motionless near the pole. Lastly, are the stars roaming independently about, yet moving in the same manner? Or is the sky rock-solid with such strength as to carry an innumerable number of stars? If so, the planets are capable of defying this motion to increasing degrees, with the Earth most defiant of all.

Sagr.: Simplicio, do the planets have their own natural motion, or are they allowed composite ones?

Simp.: The planets can only have one natural motion, but being carried by the primary mover, they participate along with its motion. Let me note that the primary Mover is all-powerful, and it is just as easy for Him to move the stars as it is to move the Earth. Look, it would be quite natural to attribute the daily motion to a rotating Earth, were this not impossible, as I will show.

I know there are some silly arguments around, such as "if the Earth's horizon is always going down in the East, then it would become easy to climb mountains in that direction", but I will stick to the most serious objections.

Firstly, the natural motion of the Earth is towards the center, as evidenced by the motion of its parts. It would require an additional eternal force to keep it rotating.

Secondly, if the Earth also has a secondary annual motion, this would imply that the stars would change their relative positions, something not seen.

Thirdly, the motion of earth and water must be towards some point, the center of the World, which only coincidentally matches with that of the Earth.

Fourthly, when one lets go a stone from a tower, it falls straight down and perpendicularly, even though the tower has moved a few hundred yards along with the Earth's motion. A cannonball fired directly upward would travel many miles westward by the time it fell.

Then again, a cannonball fired due East or West will travel the precise same amount, while those fired North or South do not deviate from a straight line.

If one were to see how fast Earth travels in one day, we would conclude that we should be feeling an overpowering easterly wind, so strong that birds ought to find it impossible to fly westwards; in fact winds and clouds do sometimes blow in the opposite direction.

At this speed, the centrifugal force would send every free object flying outwards.

Sagr.: I myself used to find the Copernican system pure madness from these accounts. Yet I started finding scholars, educated in the Peripatetic school, who changed their views when they read Copernicus. The ones who didn't had only read him superficially. Now only one of these two viewpoints can be true; and

[5] Jupiter's satellites

we know that from a false position one can find all sorts of fallacies, so I expect this dialogue to be decisive.

Salv.: Let me answer one by one. Aristotle *assigns* linear natural motion to the Earth, but we claim it is rotation. This does not mean that the parts of the Earth should rotate like it (this is like saying that every part of a sphere should be a sphere), but that every part should rotate about the center daily just like Earth does.

Secondly, we are not now discussing Earth's secondary motion; that we will do tomorrow.

Thirdly, we agree that weights move towards the center; for Aristotle this is the center of the universe, but for Copernicus, it is that of the Earth.

The fourth objection we need to analyze carefully. Tell me, how would a stone fall from a moving tower?

Simp.: The stone would have two motions, one natural and downward, the other forced on it by the tower, which compounded together do not give a straight line.

Salv.: So one motion of the stone, along with the tower, is circular, and the other downward. But if the Earth also rotates at the same rate, the stone would *appear* to fall directly down. Since the appearances are the same, the argument that the Earth is still because stones fall directly down is false. It's like letting a ball of wax fall in a vessel of water as it is moved uniformly; it would appear to fall directly down with respect to the vessel. We are like on a boat, seeing the stars moving past.

Note that I am assuming that air, along with everything else on Earth, also partakes of this diurnal rotation. This takes care of the Easterly wind, and allows westerly winds to blow and birds to fly. Similarly for the cannons fired East and West, for the cannon-ball is already moving at the speed of the cannon, which thus increases or decreases this when fired in either direction; aiming North or South does not matter as the easterly speed remains the same, neglecting the small difference due to the fact that the northerly target is following a slightly smaller circle.

Sagr.: It's like a painter inside the ship's cabin. Even if the ship had traveled a thousand miles in calm seas, the painting would have been the same as if it was made on land. The common motion of the brush, canvas and ship are not noticeable, only the relative motion of the brush and the canvas are.

Simp.: Ah, but if a bullet is let go down from a mast of a ship while this is sailing, it will fall a certain distance away from the mast, as sailors have affirmed.

Salv.: This is a different case. The ship under sail is experiencing all sorts of forces; when the bullet leaves the mast, these act only on the ship, causing it to deviate from the falling bullet. (*Simplicio utters something about evading the question.*) But to answer you better, I claim that a moving ship in calm seas would *not* have this effect on the bullet! You repeat what others have said they've seen, but I will show you directly.

Imagine a weight going down an inclined plane. How would it move down the plane, and up the plane? It would accelerate downward in the first instance, and in the second, it would decelerate, stop, then accelerate downward as before; and these the more so if the inclination is increased. What would happen if the plane is horizontal? The two answers are that it stays at rest, and that it keeps moving uniformly, as long as the plane remains horizontal. We do have such a plane, the calm sea. A ship would, in principle, move uniformly in it, and a bullet at the top of the mast would have the same speed. When let go, it would appear to fall directly down on the deck.

Simp.: Aristotelians would not accept that a stone let go from the hand would have the same speed as the ship. Once contact is lost, it would be the air that could cause it to move. I mean that a thrown stone is hurtled onwards only by means of the moving air surrounding it. So in still air, a dropped stone would be left behind the ship.

Salv.: You truly believe all this? Look, you grant me that wind affects light objects much more than heavy ones; so by your argument one should be able to throw a feather much farther than a stone, and a pendulum with a bob of cotton wool should carry on with its impetus much longer than one of lead!

And how does the air continue pushing, so to speak, the thrown stone? We can make a simple experiment where we pretend to throw something at a hanging gold leaf, and you would see that the gold leaf is raised but a little, and returns to rest immediately.

Sagr.: If I may intervene, Simplicio is saying that a bow, in shooting an arrow, sends with it the air around it. But let the bow now shoot the arrow placed parallel to the bowstring. It hardly goes far at all; yet, according to Aristotle, it would have received more "wind" than the other arrow.

Simp.: The reason is easy, the first arrow has little air to penetrate, while the second one a lot.

Salv.: But you just said that the air goes along with it! The truth is, the medium always impedes motion not help it.

In any case, suppose the air to move at the same speed as the ship, then we agree that the stone ought to fall directly down on deck. This means that it is impossible to say for certain whether the ship is at rest or moving. So this is precisely the same case as that of the stone falling from the tower.

Sagr.: But wait a minute, are you saying that the stone takes say, 2 seconds, to fall down the mast, no matter what speed of the ship? That would mean that a cannon-ball shot point-blank from the top of a tower would take exactly the same time to fall to the ground, no matter how powerful the charge is.

Salv.: Yes, that would be true, barring the effects of air resistance.

Simp.: Ok, let us put it another way: if a man on a galloping horse were to let go of an iron ball, it would reach the ground at the point where he let go.

Salv.: No, you are deceived! The ball will move some distance forward, having participated of the horse's speed. This is the same as when you let go of a ball, while moving, on an icy lake: the impetus given to it stays with it. Rather, I tell you that if the rider were to throw the ball slightly backward from the horse, it would, upon hitting the ground, move *forward*!

Sagr.: On these matters, I have seen many wonderful effects by masters of the spinning top. But, Salvatio, tell me, how does a stone let go from a tower on a rotating Earth, really move?

Salv.: To be able to answer I need to know precisely what the accelerating downward motion is; we can then compound it with its circular motion. For example, if there is no acceleration downward, then the curve would be an Archimedean spiral. If however there is a simple acceleration, as we had described in the first day, it would be some other sort of spiral. It is my belief that it moves uniformly along the semi-circle passing through the center of the Earth and the tower; thus there is no linear motion anywhere in the world, for what appears linear is in fact circular.

Sagr.: There is still the problem of the flying birds. I can understand how the air can carry the clouds in their rotation, but to push the weighty birds as well seems a bit incredible.

Salv.: The wind, though light, is powerful enough to move ships, tear down trees and even towers. The flight of the birds is like the brush we were talking of before.

Simp.: So now you ascribe the cause of motion to air! Even in calm winds, a ship is able to whiz along speedily.

Salv.: You speak like a true Peripatetic, who to see what happens in nature, looks it up in Aristotle's books. I can propose a thought-experiment that will end all this debating: imagine yourself closed in a ship's cabin; think of a flying gnat, a dripping bottle and some fish in a vessel. Now suppose the ship is moving steadily; you will not notice *any* difference whatsoever. Even a smoking incense will rise and gather at the ceiling as normal.

Sagr.: I have indeed sometimes wondered when I was below decks on a ship if it were moving or not. There remains the last objection regarding the centrifugal force.

Salv.: We all know that a bottle full of water, when swung round in a circle, will not spill the water, due to this outward force. Now, as in a sling, the direction of motion at any moment, is along the tangent of the circle. On the Earth this means that if the same thing were to happen to a stone, it would start to move from the globe's circumference along a tangent, which is what we mean by point-blank. But we have discussed this already before: we observe that any object thrown horizontally has a propensity to move downwards. All that is needed is that the downward force exceeds the centrifugal force.

Simp.: But I see that the difficulty becomes real with regard to light things, such as a feather, whose weight may not be enough to overcome the centrifugal force. Or, wait, perhaps it is a problem with heavy objects, whose tendency to be tossed out is even greater.

Salv.: It does not matter what the weight is, nor how fast the Earth's rotation is, for it is a matter of geometry. Draw the tangent at a point on a circle; its height above the circumference represents the propensity to fly off. Yet the velocities due to gravity increase linearly; they follow a straight line from the point of contact; you can see clearly that the velocities, especially close to the contact, are not as small as the distance between the tangent and the circumference. And this remains true no matter how shallow we draw the velocity line, meaning how weak is gravity, or the weight of the object.

Sagr.: Here you are assuming that the velocities do increase linearly.

Salv.: Indeed, but even if they increase as the distance of the circle to the tangent, the argument might still work up to a point. All that is needed is that they touch at the point of contact.

Simp.: These are all mathematical abstract subtleties that have no significance to the physical world. For a tangent will touch the Earth for a hundred yards or more. This reminds me of a philosopher who proved that the straight line is the shortest distance between two points, something that even Archimedes could not do. The proof is like this: let AB be a straight line, and ACB a curve; the lines AC and CB are together longer than AB, by Euclid, yet shorter than the curved parts AC and CB.

Salv.: That proof is the classic example of begging the question! I must challenge your lack of respect for the application of mathematics. It is true that a real globe may not be an ideal sphere, and a real line not a tangent, and so touch at more points, but nonetheless they are definite shapes; it would be amiss if the abstract calculations of rates did not after correspond to real coins of gold. Any errors are due to computation; even in real life, things in the abstract correspond to the concrete. I might here say that it is a lot easier to manufacture a spherical bronze ball than, say, a horse – just roll it on a circular hole in a hard material.

But to get back to business, there is another important point. It is true that the centrifugal force increases as the speed of rotation increases. But that is true for equal wheels. If the speed is increased by increasing the diameter instead, but keeping the rotation at the same rate, the centrifugal force is *not* increased proportionately. A yard-long sling can throw much farther than one of 6 yards, even if the velocity of the stone is half.

Simp.: Here I beg to differ. It is the jerk in the smaller sling that gives it its impetus; it is much harder to jerk the longer sling.

Sagr.: The question is not the jerkiness, but whether the centrifugal force of a 1-yard wheel is increased a million-fold to a million-yard Earth.

Salv.: Every object has an internal resistance to motion. It is clear that heavy objects have a resistance to move upwards, and that it is equal to the gravitational pull: one weight on a scale has an attraction downwards that balances the resistance of the other to move up. But consider how a small weight can balance a whole sack of wool in a scale of unequal arms. It manages to do this by moving much compared to the little of the sack. That is to say, a high velocity of a light object balances a low velocity of a heavy one: a pound moving at a hundred degrees of velocity resists as much as a hundred-pound moving at one. Consequently the resistance is proportional to its weight and the increase in speed.

I now claim that of two unequal wheels with the same speed at the circumference, the smaller one has a bigger centrifugal force. Two equal weights have the same resistance to the motion; the force that is causing them to move in circles must be all the more greater in the smaller circle, since the rim of the smaller wheel is retracting from its tangent more quickly than in the bigger wheel. It follows that as the diameter is increased, the centrifugal force diminishes.

Sagr.: In fact, in order that a big wheel have the same centrifugal force as a small one, one probably needs to increase its speed in proportion to its size, meaning that the rotation stay the same; but this means that there is as much tendency to fly off the Earth, as a there is from a small wheel turning every 24 hours.

Simp.: There are other objections given by two modern authors. Anti-Tycho says that if one were to take a bullet and place it at the Moon's distance, it would take more than 6 days to fall down (assuming it falls at the same speed it goes round); in the Copernican system it would fall in a spiral towards the center, but it would appear to fall directly down only at the equator and poles; at any other place it would appear to move towards the equator.

Salv.: Wait a minute, lest we rush to cut the carcass in a thousand pieces like a butcher, rather than dissect it like an anatomist. As I have demonstrated elsewhere, bodies accelerate by increasing their velocity with time, or equivalently, their distance increases as the square of the time passed, completely independent of the weight.

Simp.: This I cannot believe, for Aristotle clearly said that a heavier object will fall faster.

Salv.: Then, you would believe that a hundred-pound would fall a distance of a hundred yards before a pound would fall one yard, clearly untrue. I will discuss this later on using pendulums, but accepting my hypothesis, a weight will fall a hundred yards in about 5 seconds; so 14400 yards in a minute, and 51 840 000 (17280 miles[6]) in an hour, and 276480 miles in 4 hours. Since the distance to the Moon is 28 Earth-diameters, which is 7000 Italian miles, the bullet would take less than 4 hours to fall to the Earth.

Sagr.: I have seen how a pendulum bob will accelerate in falling then decelerate while rising in a symmetrical swing, always taking the same time for each oscillation, and these dying down presumably because of air-resistance.

Salv.: Precisely in like fashion, I imagined that a bullet let go down a tunnel that goes right through the Earth, would accelerate to the center, then in opposite fashion decelerate and reach the other side of the globe. But if the velocity increases continuously with time, say 0, 1, 2, up to some speed, say 5, then it would form a triangle, so to speak, of numbers; had it moved always with the speed 5, it would have formed a rectangle, with twice the total distance traveled as for the triangle, so that it would take as much time for a bullet to go right through the Earth as it would take it to cross half of it at the maximum speed.

You know, a pendulum will die down even without air resistance. Imagine a thread with two bobs, one midway up; since the higher one wants to oscillate at a faster rate than the lower one, it will disturb its normal oscillations. Similarly, a thread, thought of as a string of tiny weights, will go against the whole oscillation. You can see this happen clearly in a chain-pendulum, where it takes the form of an arc rather than stretched straight.

But going back to Simplicio's philosopher's objections, neither Copernicus nor anyone has maintained that a body would appear to fall down from the

[6] One Italian mile = 3000 Italian yards

Moon in a straight line. That is an Aristotelian assumption. Its motion is that compounded of its circular and downward ones.

Simp.: But do the elements, whether air or fire or earth, have a natural propensity to move in circles? If not, neither would the Earth; or is this motion forced upon it?

Salv.: Whether the circular motion is external or internal to the body, I do not commit. But if you insist upon knowing what causes the Earth to move, it is the same that causes the other planets. And for that matter, what forces the downward motion, which we call gravity? Can you explain that?

And what is the difference between intrinsic and extrinsic? It may be that what you call forced or extrinsic motion, as when an object is thrown upward, is the same as intrinsic motion, for once the object leaves the thrower it is moving on its own. Thus, upward motion is just as natural as downward.

Simp.: This can never be.

Salv.: Imagine the tunnel through the Earth; wouldn't you say that the bullet would fall through it, reach the center, then keep on going for a while? Where is the thrower that is causing it to move away from the center? Isn't it its internal impetus that causes it to keep on going? Or, if a lead ball were to fall into the sea, wouldn't its impetus cause it to fall down faster than its intrinsic motion?

Simp.: If circular motion of air is intrinsic, what would happen if the Earth were suddenly to stop, and why does it differ at the equator and at the poles?

Salv.: I guess that if the Earth were to stop, the air and the birds would continue forward; but doesn't Aristotle's heaven also have the same problems with the stars, some moving at great circles, other not at all?

Simp.: This author had other objections: imagine a huge cave at the center of the globe with a stone at its very center. Either the stone stays at rest, contradicting Copernicus when saying that objects go towards their whole, or else it moves, but then to what point, and why that and not the other? And if a stone is let go from the top of the cave, either it falls down to the center away from the whole, or it stays put, contradicting our own experiences of collapsing vaults.

Salv.: I'm afraid I do not have these supernatural experiences that this author has, but I will answer as best I can. Weights will fall towards all the parts of the Earth collectively, towards their center of gravity. So I expect that the stones would still fall down towards the center of gravity, which is the center.

Simp.: On a point of principle: how can the Earth be the cause and effect of its motion? How can a cause produce not one but three separate motions: around in a daily circle, annually around the Sun in the opposite direction, and of the axis, seasonally from North to South and back? And for what purpose when one suffice?

Salv.: I only answer that, just as animals can make diverse motions using just the circular bending of their joints, so does the Earth with only one principle. Note that the Earth rotates from West to East, if the stars are seen to move the other way; so the rotation of the Earth is in the same sense as the rotation about the Sun. There is no third motion, unless he means that the axis of rotation remains the same.

Simp.: Moreover, the Sun and the stars are fixed, but the six planets, for some unexplained reason, move. Why should planets move, but stars not?

Salv.: That the Ptolemaic system also requires an explanation for the secondary motions of the planets seems to have escaped Simplicio. But to rebut his thrust, this is another advantage of the Copernican system, in that it associates the bright splendid bodies together, and the dark ones together.

Simp.: Are the planets now dark? Is grouping the corruptible Earth with the pure Venus and Mars a step forward?

Salv.: What then is a nice way of ordering things, placing the leper-house in the heart of the city[7]?

Simp.: Lastly, even animals get tired; but it is nothing in comparison to the eternal motion of the Earth, or should I say three motions? It is not enough to say that nature sometimes tires out and loses its impetus, yet at other times continues forever.

Sagr.: It may very well be that to Earth, a rolling about is rest, just as the heart does not tire with its incessant beats. I don't mean this metaphorically only, for we tire out because we go against our natural tendencies, as Simplicio would have it, say by climbing a ladder, or raising our legs in walking. But a rotating mass, not dissipating any power in vertical motion, would not tire out.

Salv.: And only a moment ago, he dismissed the problem of a rotating starry sphere!

1.3 Third Day

Sagr.: Today we will discuss the theory that it is the Earth, like all the other planets, that move around the Sun, rather than the other way round. Simplicio, start with your objections.

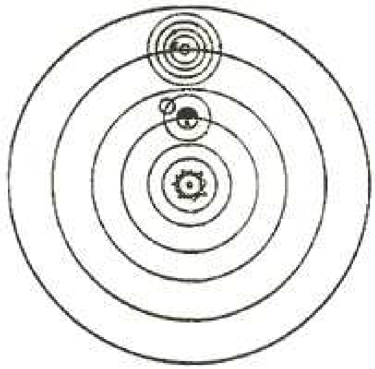
Simp.: The most obvious is that the Earth would not be at the center of the universe, as proved by Aristotle.

Salv.: You are already making a hidden assumption that the sky is finite and spherical. The only evidence he gives for this is that we see it move around.

But let me, for now, grant you that the universe has a center: does it make sense that the other planets move around a point other than the center? No, so if I prove that they move around the Sun, it follows that the center is not the Earth. To support this view, note that Venus is six times farther at its maximum distance than at its closest; Mars is 8 times; Jupiter has less variation, and Saturn even less. This we deduce by looking at their apparent size through the telescope. Moreover, Venus and Mercury are always up to fixed distance from the Sun, and they have phases just like the Moon. Only the Moon, I grant you, goes round the Earth, since it has phases and eclipses.

There are thus two inner planets, with phases, and three outer planets, always appearing nearly full. You can place the stars wherever you want, either

[7] i.e., the Earth at the center of the universe



in a sphere, or scattered in an infinite space.

Now Earth, with its yearly rotation around the Sun, would fit elegantly between Venus and Mars, whose periods are 9 months and 2 years respectively. If the Earth did not rotate on its axis, we would have 6 months of day, and 6 months of night; so we assign a 24-hour rotation on its axis.

Simp.: Even to move a boulder across a plain requires tremendous effort; how can you calmly assert that the whole immense Earth moves effortlessly such huge distances? And you depend much on the telescope with its deceptive lenses.

Sagr.: But before, I want to hear why we do not see the forty-fold change in size and brightness in Venus, as you suggest.

Salv.: Mostly, it is our eyes that deceive us when we look at small things, such as a planet or star. For it introduces a haze surrounding the central point of light, increasing its diameter tenfold or more, making it difficult to see the changes in their apparent size. To see this, bring your clenched fist to your eye so as to allow only a tiny area of vision, and look at Jupiter and the Dog Star, and you will see that the first is larger than the second, even if they appear of equal size to the naked eye. I have often seen Venus looking larger and brighter than Jupiter, yet when I look at through the telescope, it is the other way round. Moreover, in the case of Venus, there is a compensating effect, as it is horned and mostly in shadow when it close and large, but full when small and far.

We do observe the changes in the other planets through the telescope, except Mercury, which can only be seen weakly since it is so close to the Sun. Here I wonder at the resolve of Copernicus in sticking to his reasoning when he saw no such changes. He was led to his system when he tried to fit a Ptolemaic system to his observations; the result was an unwieldy mess of epicycles, some rotating this way, some the other way. So he read about other systems, including that of the Pythagoreans. This fitted the observations much better, using simple circles with one center, and all moving in the same sense.

Admittedly, the Moon would stick out as a sore point in the Copernican system, were it not for the new observations that Jupiter has not one but four moons. For that is what they are, being invisible when they go in Jupiter's shadow.

Simp.: How does the Copernican system do away with the retrograde motion?

Salv.: Easily! Take two concentric circles, around the smaller one goes the Earth in a year, around the larger one Jupiter in twelve. Thus the Earth overtakes Jupiter, and as it does so, Jupiter appears to move backward, even though it is in reality still plodding forward.

But I have a conjecture that the Sun also participates in this motion. Our Academic[8] discovered in 1610 that the Sun has spots. They appeared and disappeared while moving around it, or rather, with it, in about a month, always from right to left. They also move among themselves, sometimes dividing, sometimes colluding, as if they are clouds. He even proposed that the Sun rotates at an inclination to the plane of the ecliptic, in order to explain why these clouds sometimes move along a solar latitude, sometimes across them.

Simp.: All this does not show that the Sun is at the center and the Earth moving around it.

Salv.: Indeed, but it replaces three motions of the Sun with but one.

Simp.: Let me put forward some objections; not the ironic ones that are sometimes heard, like "if the Sun is below the Earth, then Christ rose to hell and descended into heaven."

If the Earth truly goes round the Sun, then the stars, which appear relatively fixed, must be very far away, and consequently must be huge.

Salv.: Let us suppose that a star is the size of the Sun. How far away does it need to be to appear of the 6th magnitude? Assuming that the size of a star of first magnitude is no more than 5 seconds of arc, it follows that the size of a star of 6th magnitude would be six times less, or 50 thirds[9]. For the Sun, which is half a degree in diameter, to appear this small it would need to be 2160 times farther away than now. And as the Earth is 604 Earth diameters away from the Sun, the star would be at least a million times the Earth's diameter in distance. No wonder we don't see any difference in the stars' positions from their rising to their setting.

Sagr.: So how does Tycho arrive at his large stellar sizes?

Salv.: He and others take the apparent size of a star to be about 2 or 3 minutes of arc, and their distance 3000 times as far as the Sun, which makes them of huge size. But one can measure their apparent size accurately by hanging a taut thread and walking towards it until a star disappears behind it. Using the tables of chords one can then estimate its apparent size to be 5 seconds of arc. One may even take into consideration the size of the eye's pupil, which I estimated precisely by taking a white strip of paper fixed to a wall, and an identical black strip fixed to a stick 20 yards away, and finding the place where the edges of the white strip appear first behind the black's; by similar triangles one can find the pupil's size.

[8] Galileo

[9] Degrees are divided into minutes, seconds, thirds.

Simp.: Perhaps there is no stellar parallax compared to Earth's diameter, but how is it that there is none compared to the diameter of the Earth's orbit?

Salv.: Let us suppose, as Ptolemy did, that the stars have an orbit of 36000 years. In comparison with Saturn's 30 years and 9 times the distance to the Sun, this would place the stars at 10800 times as far as the Sun. But irrespective of this, I doubt that anyone has tried to look for such annual variations in the stars; it would be greatest for nearby stars far from the ecliptic. Now the accuracy of our sextants is 2 minutes at best, what we really need is an instrument some twenty miles long; only then will one minute of arc translate to more than 10 yards. I have in mind to place a large beam at the top of a mountain, find the place where the star of the Big Dipper is just hid behind it when it is at its lowest point, and repeat the same 6 months later. It should be visibly higher, using a telescope.

Simp.: No one can deny that God can create the Universe as large as He pleased. But are we to admit that things were created in vain? For what use is that immense empty space between Saturn and the stars? And what purpose have these large stars, surely not to serve this inconsiderable dot of Earth?

Salv.: We should be content in discovering His creation, not His intention. It's like a grape thinking that the Sun was created solely for its ripening.

Sagr.: Who knows, just as we don't know what the spleen and the gall bladder are for? Anyway, how do we know space is empty? Did the Jovian moons suddenly come into existence, and the nebulae became star clusters, when we observed them with a telescope?

Salv.: And let us not forget that the ratio of the distance to the stars to that of the Moon, is less than that of the elephant to the ant. Secondly, what purpose do the stars have in the Ptolemaic system? What an unbelievably large number of faint stars, but for what?

Sagr.: It is now time for you to explain to us the third motion of Copernicus.

Salv.: We assume that the Earth rotates about a fixed axis, but moves as a whole around the Sun. This axis is permanently inclined to the plane of the ecliptic at an angle of 23 and a half degrees from the perpendicular, and the north pole pointing towards Cancer. The length of day is thus determined by the proportion of the latitude intersecting the terminator line that separates day from night.

As the Earth moves around the Sun, the terminator moves with it, and at one point, when the Sun is in Cancer, the axis tilts towards the Sun. Those on the tropic of Cancer would see the Sun directly above. Those near the north pole have a 24-hour day, while those near the south pole have no day.

When the Sun is in Libra, the pole is on the terminator and all places have equal day and night. Later, in Capricorn then Aries, the situations are reversed.

Simp.: No wonder that Aristotle faults Plato for admitting no one unless proficient in geometry. Why doesn't the axis turn around the Sun with the Earth? You must now introduce a fourth motion to reverse this natural motion of the axis, to make it fixed in space. That a single body can move in four different ways yet obey one law is beyond me.

Salv.: It doesn't. The first motion towards the center is not of the whole

Earth, but of its parts. The annual and diurnal motions are in the same sense, and so are not more contradictory than a ball rolling down a spherical slope. As to the fourth motion, it may be entirely natural: for take a floating ball in a basin of water, held in your hands, and make one whole turn; you will see that the ball will turn in the *opposite* direction, but in reality for an observer seeing the whole thing from a distance, the ball has not turned at all.

In any case, that there are preferred directions in nature is not out of the ordinary. Look at a lodestone and how it always points in the same direction; how much more if the whole Earth is a huge magnet, as found by Gilbert; and I thank the Peripatetic who gave me the book to rid his library of it! Tell me, do you think the Earth is one substance?

Simp.: Apart from the seas, it seems to me to consist of different materials: sand, soil, mountains, porphyry, alabaster, jasper, marbles, metals etc. But to me these are like jewels studded among the soil, which is the pure earth.

Salv.: That is not true, for if you dig deep you will find solid rock, as you would expect with all that weight compressing it. It is not impossible that the central material is a lodestone.

Simp.: That the rocks are compressed soil I would not dispute, but that it is of a different type of substance I see no reason to believe.

Salv.: It could perhaps be that the source of the problem is the fact that we call both Earth and earth by the same name.

Let me tell you some of Gilbert's findings. That magnets attract pieces of iron, and point to the poles, is well known. He found that it also turns downwards along a line of magnetic longitude, and the more so the more north you go, and in fact is vertical at the magnetic pole and horizontal at its equator. This agrees perfectly with a lodestone, as an experiment using a small compass and a big magnet will show. How can the Earth have the same properties as a magnet, if it doesn't consist of the same material?

Sagr.: I was impressed by how arming a magnet can strengthen it to as much as eight times. Our friend has a magnet that could lift an incredible 26 times its own weight! Where does it gets its strength from?

Salv.: I admire Gilbert as I admire more the inventor of the harp, than the hundred of artists who came later and perfected it. I have noticed that a magnet with a paper interposed between it and the arming, is less powerful. So it is not the arming itself which increases the magnetic strength, but the fact that iron is purer and denser than lodestone, which is mixed with non-magnetic stones. In fact if one places a needle on a lodestone, then bring a nail, the needle will attract to it until it touches and leaves the stone if the eye faces the nail, and leaves the nail otherwise. Also iron filings will stick only to the darker parts of a lodestone. This explains why the lodestone does not attract as much as when armed by the polished iron.

Simp.: We philosophers ascribe these to Sympathy and Antipathy between substances.

Salv.: And thereby, using this single principle, explain any occurrence in nature. This reminds me of a friend of mine who instructed his painter where to put the huntsman and the stag, and Diana, and the woods and hills, by writing their names in chalk on the canvas, and then boasted that he had actually painted it. To wit, names are not explanations!

I remember now why I started to talk about magnetism. A lodestone has three natural motions: first as a weight downwards, second as a magnet turning to the poles, and third Gilbert's tilting downwards.

Simp.: But a lodestone is a mixture of the pure elements.

Sagr.: Wait a minute, how can the earthly elements with their linear motions up and down, ever compose a circular motion? And if lodestone is mixed, how much more soil and earth!

1.4 Fourth Day

Salv.: I dare say that we have now accounted for all of Earth's motions, except for one, namely the tides. The solid earth as it rotates must affect the fluid sea. I have thought a lot about the ebb and reflux of the tides, and cannot see how it can possibly come about if the Earth is immobile.

As far as I can tell, tides have three periods: half-daily, monthly and annual. The last two seem to be secondary alterations due to the Moon and Sun respectively. The diurnal flows are of three types, according to their place: a rise/fall, an Easterly/Westerly flow, or both. It seems the last occurs when the sea ends in a beach, and the first when it terminates in a cliff; but in the open seas, as witnessed in the Mediterranean islands, the change in sea-level is minimal, but the currents considerable.

Simp.: Aristotle has written something about tides: he thought that it is an oscillation of flows between deep and shallow waters. Kepler has said that it is the Moon that pulls the sea continually towards it, as it turns around the Earth, thus causing the daily ebbs and flows; he noticed that the tide is highest right under it, but somehow also at those points on Earth right opposite it. Others think that the Moon heats the sea in some way.

Salv.: Aristotle must have forgotten that open seas vary also in depth. As to the second suggestion, this rise of the sea-level is never observed in the Mediterranean, save in Venice and the Eastern part. Finally, tell those who suggested the third one to put in their hand in a kettle on a fire until the water rises but half an inch.

The rise in sea-level in Venice cannot be due to water coming in from the Gibraltar straits, otherwise to travel a few thousand miles in six hours would give a very noticeable current. There can be two possible explanations: a whole body of water oscillates from one end to another either because the ends are alternately raised and lowered, or because the container moves irregularly, alternately accelerating and decelerating and causing the water to accumulate first on one side, then on the other, but remaining level in the middle.

It is my hypothesis that the combination of the diurnal with the annual rotation of the Earth causes a "wobble" so to speak in the sea. To explain this let us follow the sea as it rotates with Earth: on the night side it has a bigger velocity than on the day side because of the annual rotation. This variation in speed sets the seas in a daily oscillation. Now, just as pendulums have different periods depending on their lengths, so these bodies of water have theirs depending on their lengths and depths. There is also an effect due when a long sea basin has different accelerations at its head and end.

This explains various observations: why lakes and small seas have no tides, since the whole body accelerates slowly and all together; long stretches of seas in an east-west direction have high tides; although the flow in one direction ought to be of 12 hours duration, the water ebbs back in a few hours, so that it appears that there are two tides every day. Moreover the size of the body of water determines the period of oscillation, as noted before; these tides would be greater if these periods agree, and small if they cancel each other; in fact the period of 6 hours is observed in the Mediterranean, but shorter ones at Hellespont and the Aegean sea; it also explains why the Red Sea, which is long but runs north-south, is almost exempt from tides. And why tides are maximum at its ends, such as Venice; and why there are strong currents in the middle, such as the Sicilian channel and the Bosphorus. The mixing of different periods causes different flows and the great disturbance where they meet, of which sailors are wary because they cannot be detected easily. Strong winds and large rivers will also magnify or reduce the rise in level, such as in the Black Sea.

Simp.: You argue well, that the tides may be caused by the Earth's motion; of course this is a sufficient not a necessary reason. But wouldn't the air, which is much more fluid than water, also be affected in the same way, rather even more?

Salv.: On the contrary, its levity causes it to have a smaller effect, because it has much less impetus. It is easily moved by much less force, including the mountains and rugged terrain. Above the mountains, I tend to agree that air might be freer to move independent of Earth's motion. But even down here, there are large stretches without mountains, namely the oceans: and indeed, towards the equator, there is a perpetual easterly gale, which sailors use to take them to the West Indies, and even beyond in the Pacific Ocean.

Sagr.: I can also confirm that voyages from the East Mediterranean to Venice take about 25% less time than the contrary voyage.

Simp.: Yet I cannot help but notice that these winds can easily be explained by the westerly motion of the Heavens, rather than Earth's. In fact we use very much the same reasoning to explain the same winds and currents that you just noted.

Salv.: But there is a difference: you suppose that there is a crystal clear sphere holding the Moon, below which is an element of Fire; or, as others profess, that there is a pure tenuous ether beyond the Air. With your explanation, it is these tenuous substances which push the air and water; with ours, it is the mountains and land-basins, which is much more reasonable. Moreover, from one uniform motion, there cannot result a periodical occurrence. A river, ever flowing, produces a constant meander, but the tides are different.

I am more at a loss in explaining the secondary effects of the Moon and Sun. How can they, being so distant, affect what happens here? Without bringing in imagined and occult causes, such as light and heat, I have to conclude that they must be due to slight, monthly and annual, variations in the Earth's motions. This could be due to different speeds both in its annual and diurnal motions. I almost despaired of finding the causes of these: the period of a pendulum does not depend on the swing's magnitude; equivalently if you roll a musket ball down a spherical bowl, starting from different points, they reach the bottom at nearly the same time.

But watch a clock-maker how he adds little weights of lead to moderate the torsion, and makes it tick faster or slower as needed. For pendulums, the periodicity depends only on the length of the swing; and we see exactly the same thing happening in the periodicity of planets, and even of the Jovian moons. Their orbital time becomes longer with increasing radius, and we can be certain that if the Moon should get closer to Earth, its period would shorten. Now we know that the Moon is sometimes closer to the Sun, sometimes farther, than the Earth; this variation in distance causes a variation in its speed. This implies that the Earth, in trying to make the Moon go round it in a circle, must accelerate and decelerate continually, just like the clock's leaden weights.

The other annual effect is due to the axis of inclination, which at the solstices and equinoxes is inclined differently with respect to the Sun (so that the velocity of daily rotation is inclined with respect to its annual one).

Sagr.: Surely these irregularities ought to have observable effects.

Salv.: First of all, remember that we are looking for a difference of 1 in 1000. Secondly, there is much that has still to be discovered. Indeed, astronomers even now are discovering irregularities in the motions of the Moon, and the planets, including Saturn and Mercury. Ptolemy introduced epicycles and eccentricities many years after Aristotle. How each planet actually moves in its orbit, and what shape this is, is studied by astronomers in the Theory of the Planets; but these are refinements of the Copernican system. For example the Sun is observed to traverse the lower part of the ecliptic in 9 days less than the upper.

To test these effects one needs to make long and precise observations of the tides throughout the Mediterranean. For example, it would be convenient to this theory if the high tide in Venice corresponds to low tide in the East Mediterranean.

Sagr.: To recap, I find the following in favor of the Copernican system: (i) a simple explanation of the retrograde motion of the outer planets, (ii) the Sun's rotation, as attested by its spots, (iii) the tides are explained naturally. We may perhaps find that stars do have parallaxes, and recently there is this study of Cesare Marsigli of the change in the meridian that may also prove in favor.

2 Mathematical Discussions and Demonstrations on Two New Sciences Mechanics and Kinetics (1638)

2.1 First Day

Sagr.: It is the common opinion that machines which work on the small scale should work on the larger one, seeing that mechanics is based on geometrical figures, whose properties are independent of size.

Salv.: That is absolutely false; large machines are more accurate, but also weaker. To take a simple example, consider a pole fixed perpendicularly into a wall, just long enough to support its weight, say, a hundred times long as it is wide. A shorter pole of the same proportions will be stronger, while a longer one will break. This is not so unfamiliar, for we all know that a horse will break a leg if it falls a few feet, but not a dog or cat, and a grasshopper can fall a whole tower without injury; and that a large column is much more fragile, when resting horizontally, than a small one.

Sagr.: Yet a nail twice as large as another will hold more than 4 times the load.

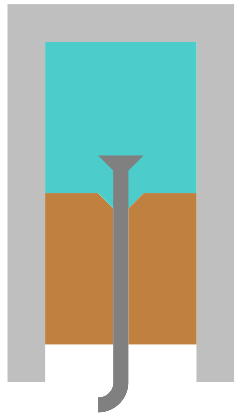
Salv.: Imagine a vertical cylinder from which hangs a weight. If the latter is big enough, the cylinder will break; there is a maximum resistance that is proportional to the cross-sectional area. One finds that concrete is stronger than wood, although the latter have fibres running along their length. A rope is just as strong as a wooden cylinder because its fibres, though short, intertwine with each other. For example, a rope wound round a pole will tighten itself the more you pull, even if one end is free.

As to where stone and concrete get their strength, some say it is due to the abhorrence of introducing a vacuum. For example take two polished slabs of marble or glass, placed on top of each other; it is easy to slide them but one can hold the upper slab and the lower does not fall from it. But two rough slabs can easily be separated because air can enter. Here a difficulty presents itself: how can the vacuum cause the slabs to 'stick' when it has not yet formed? Nevertheless, the vacuum is not the only cause of a material's strength, except for liquids like water.

How shall we separate the vacuum force from the other? Consider the following experiment: first invert the apparatus shown, fill the bottom part with water, then close the iron wire and invert again; the wooden plug will not fall, and will hold a maximum weight equal to the force of the vacuum. In practice air will not enter above the water, if the wooden plug fits tightly and is oiled.

Sagr.: This experiment reminds me of a pump that had stopped working. The repairer told me there was nothing wrong with it, but that no pump can pull water more than 18 cubits. Now you make me realize that this is like a rope which is long enough to break under its own weight.

Salv.: Exactly, and this height of 18 cubits is independent of the diameter of the pipe. It allows us to easily calculate the force due to vacuum: it equals



the weight of water 18 cubits high with the same diameter.

Let us now do some experiments to determine the real strength of a material, subtracting away the vacuum force.

Sagr.: What is the glue that unites stone or glass? No real glue can survive the furnace, yet glass or gold will melt and then reunite as if nothing had happened.

Salv.: I can only offer a tentative explanation: it may be that solids are held together because of a large number of tiny “bubbles” of vacuum trapped inside; fire manages to fill these, causing the metal or glass to melt and separate; but when the fire particles escape them during cooling, the vacua reform.

Let me digress to talk about the possibility of having an infinite number of such vacua in a finite body. Consider a regular polygon standing on one side, and containing another concentric polygon; suppose it rotates on its sides. As it does so, the outer polygon traces out a line equal to its perimeter, while the inner polygon traces out a line with gaps, in total almost equal to the perimeter. Now suppose the polygon’s sides are so much increased in number that it is practically a circle. The outer circle rolls out a perimeter, while the inner one also traces out a line of the same length; how can a circle’s circumference yield a length of a circle twice its size, unless it has gaps? There is no slipping, else an infinite number of such finite slips would give an infinite line; and there is only one point of contact throughout the motion. The line is not a continuum but contains an infinite number of tiny indivisible vacua. This allows the expansion of a body without the introduction of empty spaces.

We must be careful, however, for what are we to make of the center of the circle which, alone, traces out a line of the same length? Let me show you the following demonstration. Take a cylinder, whose diameter is twice its height, and carve out a hemisphere; and consider also a cone with the same circular base and height. Let us cut both solids at a certain height. One can show that the volumes of the upper parts, as well as their base areas, are equal. But as we increase the height, they tend to the circle and point respectively, so that we are justified in saying that the circle and the point have the same magnitude.

Sagr.: But if one line contains an infinite number of points, then twice that line contains twice infinity.

Salv.: Our finite minds find difficulty grappling with the infinite. But one infinity cannot be greater than another. To demonstrate this, there are more numbers than squares, for some are non-square; but there are as many squares as there are roots; yet every number is a root. Similarly, lines of different lengths both contain the 'same' infinite number of points.

A divisible cannot be constructed out of a finite number of indivisibles, else the act of division causes a partition of the divisible. A line must therefore consist of an infinite number of immeasurably small elements. Now a finite line cannot contain an infinite number of finite parts; yet it can contain as large a number as we want. We can perhaps say that it contains a potential infinity of points; not actual as no one can actually do it. One may try to repeatedly bisect it, but no matter how long you wait, there are of course still a finite number of divisions. Rather, one recedes more from infinity, for a large number contains a less percentage of squares. To see it another way, by dividing a line into a finite number of equal parts, one can bend it to form a polygon; in the same way bending at an infinite number of points gives the circle; one can 'see' the points as the circle wheels round on a line.

In the same vein, there are as many circles as there are points in a line. For take a line AB, and any point on it that divides it into AC and CB. The locus of points whose distance from A and B are in the same ratio as these lengths is a circle[10]. Varying C gives circles of any radius. But when C is the midpoint, the circle "becomes" a straight line, which can be thought of as an infinite circle.

In my view, melting is the passage of a solid body into its infinitely small indivisible components. Instead, filing a solid gives a large, but finite, number of particles; the properties of the powder heap are different from that of the fluid or solid e.g. compare ground glass with the transparent solid.

Sagr.: I have seen how a spherical mirror can reflect the sun's rays and melt lead. The works of Archimedes and Father Cavalieri both attest to this power of light.

Salv.: Yes, light must have some internal power of swift motion, just as lightning and gunpowder are accompanied with blasts; and charcoal fire rages more with bellows.

Simp.: Everyday experience shows that light travels instantaneously; for an artillery flash reaches our eyes at the same time as it explodes, but its sound comes later.

Salv.: That only tells us that light travels faster than sound; you can't tell whether the sun reaches the horizon before you actually see it. Once I devised an experiment to see if light has a finite speed. Take two persons with lanterns; let them practise covering them with their hands and when one takes his hand off, the other does the same; they soon acquire the skill of doing this very quickly; now let them separate by say 3 miles at night (or 8 miles using telescopes) and repeat; I found there was no sensible difference, so that light travels either instantaneously or with extraordinary speed (like lightning for example).

[10] Galileo gives a geometric proof of this

But to go back to our original discussion on the line, let me say something about how it explains not only expansion but also contraction. For, just as the smaller circle with a lesser circumference traces out the same line, a bigger circle with a larger circumference traces out the same line, and this is a contraction. In this case, there must be superpositions instead of vacua. It's either this or there are real empty spaces between the particles of a body, which has its own problems in that it would not explain why objects do not go through each other.

Simp.: According to this view, an ounce of gold may be expanded to the size of the Earth, and the Earth may be contracted to a walnut. I don't believe your abstract mathematical arguments apply to the concrete reality.

Salv.: Do you know how gold wire is produced? They take a rod of silver the diameter of a few fingers, then they hammer on a number of very thin gold leaves; this rod they force through thinner and thinner dies, until in the end it is as fine as a hair. Imagine how much the original gold has been expanded.

Simp.: Not at all! The gold's length may have multiplied, but its depth reduced.

Salv.: That cannot be, for the outer surface area of a cylinder (not including the bases) is proportional to the square root of their length, keeping the volume fixed. For the volume is proportional to the square of the diameter times the length, while the surface area is proportional to the diameter times the length. The gold leaves must have expanded tremendously in area; if it kept its volume, its thickness would be an incredible one twentieth of that of a gold leaf.

By the way, the volume of cylinders with the same surface area (without the bases) is inversely proportional to their heights; the proof is the same. This explains the following proposition, which common people always find perplexing: a cylindrical sack made from a rectangle of cloth holds more if the *longer* side is made into the base.

Sagr.: I bet most people would think that bodies with the same surface area would have the same volume. Similarly, people often measure how big a city is by walking around it, yet the perimeter bears no relation to its area. Sacrobosco shows that the circle has the largest area among polygons of the same perimeter.

Salv.: In this regard I have the following proposition: it is obvious that a circumscribed polygon has a larger area than the circle; but also a circle has a larger area than any regular polygon with the same perimeter. I can show that, as the number of sides increases, the circumscribed polygons decrease in area, while the isoperimetric polygons increase; and that the area of the circle is the mean proportion[11] of the areas of the circumscribed and isoperimetric regular polygons with the same number of sides.

The proofs follow from the following: the area of a circle is the same as of a triangle whose sides are the circumference and the radius; that of the circumscribed polygon is also the same as of a triangle with sides equal to the perimeter and the radius; it follows that the area of the circle is that of the triangle on the perimeter of the isoperimetric polygon and the radius of the circumscribed one, two polygons that are similar to each other. To show that

[11] geometric mean

increasing the number of sides decreases the area, let triangles OAC and OAD represent half the sides of circumscribed polygons; let the circle cut their lines at A, G and F; pass another circle through C, cutting OA extended at I, and OD at E; then the area of OCD is bigger than of OCE, while that of OAC is smaller than OAG; but the sectors OIC and OCE are in the same ratio as OAG and OGF, so that $OCD : OGF$ is bigger than $OAC : OAG$.

We were trying to see how it is possible for expansion to occur without introducing vacua. Imagine what big expansions occur when gunpowder flares into fire. We always see wood burn into fire, and flowers or water dissolve into odors, but never the other way round.

Aristotle argued against the vacuum, stating that heavier objects move faster than lighter ones in a given medium, and faster in a rare medium than a dense one; so that a vacuum would imply instantaneous motion. Now the former is not true, for I have tried the experiment with a cannon ball and a musket ball from a height of 200 cubits. But I also have a logical argument against it: suppose that, indeed, a heavier object moves faster than a light one; then joining them together, the heavier is held back by the lighter; so, considered as a single object, it is heavier yet would move slower.

Simp.: But the lighter object will help it to fall faster, by increasing its weight.

Salv.: You are mistaken, for how can one object press on another when they are falling at the same speed? It's like trying to lance a man when he is running away at the same speed.

Simp.: Surely you're not saying that a grain of sand falls as rapidly as a grindstone!

Salv.: It is much closer to the truth than saying that motion is proportional to weight. Of course the medium affects the fall, just as the same gold in the form of a ball or of a leaf will fall differently.

His second assertion is just as false, for it implies that every object that falls in air will fall in water, albeit more slowly. Yet a wooden ball floats.

Simp.: His assertion was about objects that fall in both media.

Salv.: You are making it worse. Suppose the ball falls with a speed of 10 in air, then it ought to fall with a speed of 1 in water; as this does not happen, you need a heavier weight which actually falls with a speed of 1 in water, and so of 10 in air. You end up with two weights that fall at the same speed in air.

How could all the philosophers fail to notice that a marble egg falls in water so much quicker than a hen's egg, yet take almost the same time in air? I conclude that a large vacuum may not exist in practice, but not in principle, and that these arguments do not preclude the existence of tiny vacua.

Sagr.: In stating that objects fall in the same time, you are assuming they have the same density; and what do you replace Aristotle's law concerning the resistance to motion?

Salv.: I have thought a lot about this. Experiments show that, indeed, heavier objects do fall faster in water; and the difference is more pronounced the denser the medium. Yet they fall at the same time in air, and some, such as roots and certain woods, neither sink nor float.

Sagr.: I once tried to add sand grains to wax until that happens, but I grew impatient.

Salv.: Nature surpasses us once again: fish can do this easily by changing the amount of air in their bladder. Physicians test the specific gravity of waters as follows: place, say, sea water at the bottom of a vessel, and fresh water on top of it; a ball of wax will then stay in the region between; if pushed down it will rise, and if pulled it will sink. It is found that the specific gravity of water is very sensitive to the addition of heat and cold water. The speed of an object is thus not due to the resistance of separating the medium.

Sagr.: But if there is no coherence in a medium, why does water form drops and not spread out evenly?

Salv.: I don't know why, but since it is not from coherence, it must be an external force. If there is coherence, a drop of water ought to stay intact when a heavier liquid like wine is poured in around it; in fact as soon as they touch, it will spread into it. There seems to be an antagonism between water and air, for take a small glass globe with a small opening at one end; fill it water and turn it upside down, and the water will not rush down and air up; now put it into a glass of red wine, and you see streaks of red going up into the globe, until all the wine ends up there. Why this happens is a mystery to me.

Gold will sink in quicksilver, but not any other metal or object, yet they all take the same time to fall in air. My conclusion is that in a vacuum, all objects would fall with precisely the same speed. To support this claim, we need two objects of different weight, but otherwise identical, and let them fall in air. I did this with a leather ball and a ball of lead, whose difference in weight was more than a thousand-fold; yet the speed of the latter was not twice of the other. Thus the difference is not due to the difference in specific gravity but due to air resistance.

Simp.: Yet the air resistance has not changed in the two cases.

Salv.: A body falls towards the center of gravity of the Earth by accelerating, i.e., by increasing its velocity with time, assuming no external factors. The medium offers a resistance proportional to the speed. In effect, the velocity increases until the resistance is so great that there is a balance.

It is well known that the effect of the medium is to reduce the weight of the object by the weight of the medium. To improve upon Aristotle, we may suppose that the medium detracts from the speed of an object an amount inversely proportional to its weight. For example, lead, which weighs ten thousand times as much as air, will have its speed reduced by 1 in 10000; while in water its speed is reduced by a greater amount. Neglecting the tiny air weight, we can say that the ratio of the speed of falling in air to water is equal to the ratio of the weight of the object to its excess weight in water.

Sagr.: I'm curious to know what the specific gravity of air is.

Salv.: That air has weight is clear from the fact that a leather bag full of air weighs more than the same empty, as Aristotle attested. In fact, it is not clear whether there are *any* substances that do not have weight. To measure the specific gravity, I took a large flask with a valve attached to the neck; I added as much water through the valve as I could, the water compressing the air inside; then I weighed it, even adjusting the weight with sand, opened the valve and reweighed by removing some sand; the ratio of the weight of the sand to the weight of the water is the specific gravity, which I found is 1 in 400.

We still need an experiment that confirms that different weights fall in equal times in air. The difficulty lies in choosing the height: too small leaves room for doubt, too high and the small air resistance slows the lighter one. It occurred to me that we may slow down the speed by letting them fall along small inclinations. Accordingly I took two identical balls, one of lead, the other of cork, and let them swing as pendulums; even after a thousand swings, they were still in perfect synchrony. The effect of the air resistance was to decrease the amount of swing of the cork ball, but its period stayed the same as that of the lead ball.

Simp.: So, in fact, the speed of the lead ball is higher than that of the cork!

Salv.: But if you take a cork ball with a swing many times larger than of the lead ball, the periods are still the same. This means that the time it takes for either the cork or the lead ball to fall a given angle is the same.

Sagr.: Now explain how a cannon ball falls faster than a bird-shot, though made of the same material. Similarly, fine powder takes a long time to settle.

Salv.: I think that the surface of objects while moving in air produces a resistance. A solid on a rotating lathe, or a top, produces a buzz which diminishes with the speed. The smaller the body is, the bigger is the ratio of its surface area to its volume, and so the bigger the air resistance.

Simp.: Wait, you mean the larger body, having a bigger surface area, should be slower.

Salv.: It is the ratio of the resistance to the weight that increases, not the actual resistance. The resistance becomes equal to the weight at a lower speed for smaller objects.

It is my belief that every object, no matter how heavy or large, will reach a maximum speed; even if thrown forcefully at a higher speed, it will slow down due to air resistance until it reaches this speed. For example, a cannon ball slows down upon hitting water, and softly touches the sand below; similarly the shot from a gun fired directly down from the top of a tower hits the ground with less speed than when it is fired from a short distance up.

Moreover, it is likely that, without air, the momentum of a body at the time of hitting the ground is the same at that needed to throw it to its original height. For observe how in a pendulum, lifting the bob a certain angle will cause it to swing up the same amount. But when air is present, the blow on hitting the ground is much less than the force needed to throw it up to its height.

I will now divert to some questions pertaining to music.

Sagr.: Good, for I have never understood how some combinations of notes are pleasing, others jarring. Also a vibrating string will set an adjacent identical string vibrating as well.

Salv.: The period of oscillation of a pendulum is proportional to the square root of the length. One can in fact measure the length of a long pendulum by counting the number of oscillations, and compare it with that of a small pendulum of known length.

Sagr.: I often observed in churches how the lamps, hung from great heights, keep a perfectly regular time. It did not square with the theory that it is the air that keeps them in motion.

Salv.: The period is fixed by nature; take a pendulum in your hand, and try to make it go faster or slower; you will not be able to. Conversely, one can set in motion even a heavy pendulum by blowing on it at the rate of its frequency, just as a single man can make even the heaviest church-bells ring. This explains how one string can set another one vibrating. In fact it does so even if it is an octave higher or lower. What is happening is that the first string sets the air vibrating, and this in turn sets the second string in motion, and any other object that has the same frequency (and no other), such as a wine-goblet.

Sagr.: The usual explanation for the octave being a difference of half the frequency has never convinced me. It is based upon the fact that you need to halve the length of a string to get it. But then again, you can also achieve it by quadrupling the tension of the string, or reducing the size of the string by four. Similarly to get a fifth, one needs lengths in the ratio of 2:3, tensions in ratio of 4:9, and sizes as 9:4.

Salv.: I have seen how water in a musical wine-glass produces waves; it sometimes happens that the note suddenly increases by an octave, and then I observe that the waves double in number. But these are so short-lived, that I made an invention to better observe them. By accident I found that when I scraped on a brass plate with a chisel, it would make a whistling sound as the chisel left periodic marks on it. The pitch depended on the speed of the chisel. Later I found that some strings of a nearby harpsichord vibrated in unison. So I did an experiment to find which notes corresponded to which chisel spacings, and it turned out that an interval of a fifth is in fact 2:3.

Note by the way, that it is not the size of the string that matters, but its weight, and this for the same reason that it is weight that impedes motion, not size. But the point is that an interval is determined by the ratio of the frequencies, not the length or tension of the string. This is why some combination of notes are displeasing, because their vibrations hit the ear tympanum in discordant times, while others are pleasing when the waves fit regularly with each other.

The eye can see what the ear hears, by swinging two pendulums having periods in the ratio 1:2, or 2:3, etc.

2.2 Second Day

Salv.: To return to the strength of materials, let me note that objects resist pulling much more than bending. I have studied how the bending strength depends on the cross-sectional shape of the beam.

I wish to prove the principle of the lever, first deduced by Archimedes, in which the force is inversely proportional to the distance from the fulcrum. I start by assuming that a rod, suspended at its two ends to a lever with center C, is in equilibrium. Cut the rod at an arbitrary place E, supporting the new ends by two new threads, to maintain equilibrium. Now the two pieces can equally be suspended from their midpoints G and F; and one can replace the two by equivalent weights; but the length GF is half the total length of the rod, so that GC equals EF, and GE equals CF; hence $GE : EF$ as $FC : CG$, which is essentially the lever law.

We can apply this to a beam projecting perpendicularly from a vertical wall. The arms of the lever are to be thought of as the projecting part, balanced by the thickness of the beam at the wall. Thus the weight at its end in ratio to the wall's resistance, is the same as half the beam thickness in ratio to the length (strictly speaking, one must add half the weight of the beam itself).

This explains why a ruler can hold much heavier weights when placed on its sharp edge than when on its flat face. Also the greatest load that a beam can hold at its extremity (the strength of the beam) diminishes as the length (for the moment increases proportionally, while the beam width and resistance remain the same), and it increases as the cube of the beam's thickness (since the moment of the weight remains the same but the resistance increases with the area times the height). It follows that for similar beams, the moment (i.e., the product of the beam's weight with its length) grows as the $3/2$ power of the resisting power at the base (since, equating moments, the resistance times half the height equals the volume times half the length; so the resistance is proportional to the volume; but the resistance increases with the area of the base).

Simp.: I would have thought that, being similar in all respects, the ratio of the force to the resistance would stay the same.

Salv.: I used to think so as well, but the more I observed, the more I was convinced of this. For, as we said before, tall men are more apt to injury than small children. In fact, any beam can be enlarged only up to a maximum amount before it breaks under its own weight.

Proposition: If the columns, one of width AB and length AC, the other of dimensions DF and DE, are at breaking point, then the ratio $DF : AB$ is equal to $DE : I$ where I is the third proportional[12] of DE and AC.

It is plain that it is impossible, whether in nature or in art, that a structure be increased indefinitely in proportion. Ships, palaces, trees and animals have a maximum limit, keeping the same design. One must use stronger or thicker materials. A dog can probably carry two of its own on its back, but I doubt whether a horse is able to carry even one.

Simp.: I'm still not convinced, since whales can reach much bigger proportions than an elephant.

Salv.: You are right, but in water fish have no weight. The case is reversed, for land animals it is the bones which have to carry their own and the flesh's weight, while in aquatic animals, it is the flesh which buoys up the heavier skeleton. Indeed, a whale is helpless on land, and a loaded ship would probably break of its own weight if dragged ashore.

Up to now we have discussed beams driven in a wall. Let us now consider a beam on a fulcrum: it clearly can have at most twice the maximum length of

[12] i.e., AC^2/DE

a wall-beam; the same thing applies to a beam supported at its ends, for the same reason that the maximum bending force occurs at its midpoint. A harder problem is to determine the maximum load that can be applied to a point on a beam supported at its ends, when the point is not in the middle; e.g. when a stick is broken on the knee, not held at its center.

To this end, consider a beam supported on a fulcrum, with arms DF and FE. It will break when the resisting force over at the fulcrum F is equal to the same resisting breaking force when the fulcrum is at the midpoint C. The force applied at D has to be increased in the ratio $DC : DF$, which can be increased indefinitely, while that at E is decreased in the ratio $EF : EC$ (which is at most 2). Thus the total force is arbitrarily large, as the fulcrum approaches one end. More precisely, the total force is proportional to the rectangle $DF.FE$ (for the ratio of forces $D : D + E$[13] is equal to $FE : DE$). A weight that is larger than the maximum possible central load (by an amount w), can still be withstood if placed at a point towards the endpoint; more precisely at the point R, where $CR = DC\sqrt{1 - 1/w}$).

Now if the beam is not a prism, it may have different properties. For example, if a rectangular beam is cut diagonally (lengthwise) to give a triangular beam, the resisting moment in the triangular beam decreases with the area (i.e., linearly); in the other triangular half, it increases, so it stands to reason that there must be a cut which would give a constant resisting moment. This path is a parabola with the vertex at the free end, using this proposition: if two levers with arm-lengths x, y and a, b, such that $x/a = (y/b)^2$, and the loads satisfy $Z/C = y/b$, then the forces are equal. Since the area under the parabola is a third of the rectangle (as shown by Archimedes and Valerio's great book on the centers of gravity), a whole third of the beam can be cut away, yet still retaining the same strength throughout the beam.

I wish to say something more about hollow cylinder beams, which are often used in nature and art e.g. bird bones, straw tubes, and lances. If a hollow cylinder has the same volume as a solid one of the same length, then their resisting moment is proportional to their diameters (since their cross-sectional areas are equal). More generally, the bending moment of a hollow cylinder is proportional to $R\sqrt{R^2 - r^2}$ where R and r are the large and small radii.

Simp.: I begin to understand that logic, though excellent for discourse, does not compare with geometry in its sharp distinctions.

2.3 Third Day

Salv.: By uniform motion, I mean that equal distances are moved in any equal times. The following axioms follow:

For the same uniform motion,

1. longer distances are traversed in a longer time.

[13] Like most mathematicians of his generation, Galileo used geometric notation, not the shorter algebraic one used here.

2. longer times are needed to traverse longer distances.

   For the same time interval,

3. longer distances are traversed with a higher speed.

4. higher speeds are needed to traverse longer distances.

Proposition 1[14]: For uniform motion, the ratio of the time intervals is the same as the ratio of the distances covered.

Proposition 2: For the same time interval, the ratio of the distances covered is the same as the ratio of the speeds. Conversely, if the ratios are equal, then so are the time intervals.

Proposition 3: For the same distance covered, the ratio of the times taken is the inverse of the ratio of the speeds.

Proposition 4: If two particles have uniform motion, then the ratio of their distances covered is the compound ratio of their speeds with the time intervals.

Proposition 5: If two particles have uniform motion, then the ratio of the time intervals is the product of the ratio of their distances with the inverse ratio of the speeds.

Proposition 6: If two particles have uniform motion, then the ratio of their speeds is the product of the ratio of their distances with the inverse ratio of their times.

We now turn to accelerated motion. When I see a stone increasing its speed as it falls, I see it doing so in the simplest manner, which I call uniformly accelerated: at any time interval, it increments an equal amount of speed.

Sagr.: As such it is just a definition; one must show that it corresponds in reality to a falling object. For example, it would imply that at shorter and shorter time intervals at the beginning of the fall, the speed is less and less. Yet we observe objects to suddenly acquire speed.

Salv.: I used to think in the same way, but then I did a little experiment. Take a heavy metal ball and let it fall onto a soft wood; decreasing the height of fall makes for smaller impressions. Similarly when a weight is driven onto a stake, it hammers it in to lesser extents as the height is reduced. After all, why should it suddenly jump from rest to a speed of, say, 10, and why 10 not 3? And when we throw an object up it slows down continuously until it does not rise anymore, while a fall is the same motion in reverse.

Simp.: But if the body spends even a moment at a speed, each time getting slower, it would never reach rest.

Salv.: The body never spends any time at any given speed, only an instant. When an object is thrown upwards, it starts with a forced impetus greater than the weight downward, but with time it loses this upward impetus until it reaches zero at the maximum height, then continues to lose it as gravitation wins over.

Sagr.: As the impressed force is reduced, there is less impetus to lose and the height achieved is less. Finally if the impressed force is just enough to balance the resistance of gravity, the object stays in the hand.

[14]Geometric proofs are supplied for each subsequent proposition

Salv.: There are various theories about why the acceleration downwards occurs: some say it is towards the center, others that it is due to the air or water which closes in behind as it opens in front, yet others that it is due to the repulsion of its particles. Irrespective of this, I am more interested in the properties of uniform acceleration.

Simp.: As far as I can see, you could have taken the definition to be that motion whose speed is proportional to the distance traversed, which is close to what Aristotle would have claimed, and to what we actually see.

Salv.: That is a contradiction, for if it acquires a speed of 2 after falling one unit of distance, and 10 after falling 5 units, then it would have taken half a unit of time in each case. It would be gratifying if the learned accepted these errors, but to hold onto past mistakes for the sake of lowering the esteem of someone who discovered a fallacy, is unbearable.

After the definition, I make one assumption: that *a body that falls an inclined plane acquires a speed that depends only on the fallen height* (provided there are no resistances, the surfaces of the plane and body are perfectly hard and straight/round). To support this claim, consider the case of a pendulum: not only does it rise up on its upswing as much as its initial height, but it does so whenever a nail is inserted anywhere inside the area of swing; although the upswing is not identical to the downswing, the height reached is the same. Unfortunately we cannot repeat this demonstration for planes, since when a ball hits the bottom of a plane it loses some of its impetus.

Proposition 1: The time taken to traverse a given distance is that time needed for uniform motion to cover the same distance with a speed equal to the average of the initial and final speeds of the uniformly accelerated motion.

The reason is that the area under the trapezium, being the shape of a linear increase in speed, is the same as the area of a rectangle with height given by the average of the end-lines. But this area is precisely the product of the speed and the time taken, namely the distance traveled.

Proposition 2: The distance traveled for uniform acceleration started from rest is proportional to the square of the time.

The distance traveled is proportional to the area under the triangle of velocities, which is proportional to the square on one side, namely time.

It also follows that, under the same conditions, the distance is proportional to the square of the final speed; and two times are in the ratio of a distance and the geometric mean of the distances.

Corollary: Starting from rest, the distances traveled in subsequent equal times are in the ratios 1:3:5:7:9 etc. (for these are the differences between squares.)

But I need to confirm that real bodies accelerate uniformly, for this is the main supposition upon which the rest of the work depends. To this effect, take a plank of dimensions 12 cubits by half a cubit; on its edge was cut a groove, polished and covered in parchment; one end of the plank was inclined by one or two cubits, and a round bronze ball rolled down the groove; I noted the length and time of descent, repeating the experiment several times and taking the average; to measure the time I took a tank of water with a pipe, which I inserted into a glass as long as the ball was in motion, and then weighed the water; I then repeated using only a quarter of the length, and found that it took half the time; this was repeated for other fractions of the length, and other inclinations, and always it turned out that the distance was proportional to the square of the time.

Let me now see how the velocity depends on the inclination. If one simply inclines the plane more, the final momentum will increase, with the maximum occurring when the plane is vertical; when the plank is horizontal, the ball has no tendency to move or to resist motion. To measure what is the downward impetus on a ball trying to fall, I found what is the least force needed to keep it at rest; this I did by tying the ball with a string, passing it over a pulley and tying it to another weight. This I do because the inclined motion can be thought of as having a horizontal component, that does not resist motion, and a vertical component, that is equivalent to a free-fall. In fact the ratio of the two weights are as the ratio of the height to the plank length, which are the distances traveled by the respective resistances. Thus the effective impetus down a slope is only a fraction of the weight.

Accepting this, we can show that the final speed depends only on the height. For let the plane be AB inclined to the vertical AC; then the impelling force is the fraction $AC : AB$ of the weight; so during the time a particle would fall AC and achieve a speed v, it would fall only $AD = \frac{AC}{AB}AC$ and reach a speed of $u = \frac{AC}{AB}v$; but we have seen that the distance traveled, AB, is proportional to the square of the speed, so the square of the speed at B is $\frac{AB}{AD}u^2 = v^2$.

Proposition 3: The ratio of the time required to fall an incline to that in vertical fall, is equal to the ratio of the length of the incline to the height; and more generally, the time taken down planes, each having the same height, is proportional to the length of the incline (since the time is proportional to the d/v, but v is the same).

Proposition 4: The times of descent along planes of the same length but of different inclinations are in the inverse ratio of the square roots of their heights. (Compare the times of descent to the vertical fall.)

Proposition 5: The time of descent down a plane is proportional to the product of the length and the inverse of the square root of the height.

Proposition 6: If from the highest or lowest point in a vertical circle there be drawn a chord, the time of descent along it is equal to that along the vertical diameter. (Since the height is proportional to the square of the length of the chord.)

Corollaries: The time taken down any such chord is the same; the locus of starting-points for which a particle takes equal times to fall down a line to a fixed point, is a circle; the times of descent down two planes will be the same when the unit vertical heights[15] have the same ratio as the lengths.

Sagr.: So, if particles start off together from one point, moving along rays with uniform speed, they will lie on expanding spheres. And if they move along rays with uniform gravitational acceleration, they will also form spheres, this

[15] i.e., $\sin\theta$

time with a common tangent.

Simp.: These wonderful results about spheres leads one to think that there may be some great hidden mystery, related to the creation of the spherical universe, the seat of the first cause.

Salv.: We simply procure the marble out of which a future gifted sculptor will carve out the masterpieces hidden in this rough shape.

Proposition 7: If the heights of two inclined planes are to each other in the same ratio as the squares of their lengths, bodies starting from rest will traverse these planes in equal times.

Proposition 8: The time of descent along a chord of a vertical circle, is more or less than that down the vertical diameter according as the chord does or does not intersect this vertical diameter.

Proposition 9: If through a point C on a horizontal line AB, two lines CD and CE are drawn, inclined at any angles, and at any point D the angle CDF is marked equal to the angle BCE, so that the line DF cuts CE at F and CFD is equal to ACD; then the times of descent down CD and CF are equal. (Use similar triangles.)

Proposition 10: The times of descent down inclined planes of the same height are in the ratio of their lengths, even if the particles start off with some velocity[16]

Proposition 11: If motion down a plane starts from rest, then the ratio of the times of descent along an initial part AB to that along the remaining part BC is the same as the ratio $AB : (\sqrt{AB.AC} - AB)$. (Can consider a vertical fall.)

Proposition 12: Let AF and DC be two horizontal planes, with AC vertical and the inclined plane DF intersecting it at B. Let AR be a mean proportional between the entire vertical AC and its upper part AB; and let FS be a mean proportional between FD and its upper part FB. Then the ratio of the time of fall down AC to that down AB plus BD is equal to the ratio of the length AC to $\sqrt{AC.AB} + DF - \sqrt{FD.FB}$.

Proposition 13: Given a vertical line AC, and a point B on it, it is required to find an inclined plane BD, with CD horizontal, such that a particle, having fallen from rest along AB, continues along BD in the same time which it took in falling AB.

Proposition 14: Given an inclined plane AC, to find that height DA directly above A, through which a body will fall from rest in the same time which is required to descend AC after falling through DA.

Proposition 15: Given a vertical line AB and a plane DBC inclined to it, it is required to find that length BF on AB extended below B which will be traversed in the same time as BC, each of these motions having been preceded by AB.

Proposition 16: If the time required for a body, starting from rest at A, to descend the inclined plane AB and the vertical line AC, is the same, then a body, starting with some speed, will take less time to descend AB than AC.

[16] Galileo does not state this directly; instead he says "if they start their fall higher up but from a constant height.

Corollary: The vertical distance covered by a freely falling body, starting with some speed, and during the time-interval required to traverse an inclined plane, is greater than the length of the inclined plane, but less than the distance traversed on the inclined plane during an equal time, starting from rest.

Proposition 17: A body falls from rest a vertical distance AB in a time t, and continues to descend along a given plane BF; it is required to find how far it descends along the plane in the time t.

Proposition 18: A body falls from rest a vertical distance AB in a time t; given a smaller time $t_1 < t$, it is required to locate points CD on its fall with length equal to AB through which the body passes in a time t_1.

Proposition 19: A body falls from rest a vertical distance AB in a time t; it is required to find the time in which the same body will fall an equal distance chosen anywhere in the same vertical line.

Corollary: If AB represents the time of fall, from rest at A, through the distance AB, and SA is an additional height, the time required to traverse AB, after fall through SA, will be the excess of the mean proportional between SB and BA over the mean proportional between BA and AS.

Proposition 20: A body falls from rest a vertical distance AB in a time t; given a point D, further down from A, it is required to find that point C such that the body takes the same time t to fall CD.

Proposition 21: If a body falls vertically (or inclined) from rest at A to C in a time t, and continues along an inclined plane CG, then the distance traversed along the inclined plane, during the same time t, will be greater than twice, and less than three times AC.

Proposition 22: A body falls from rest a vertical distance CD in a time A; it is required to find a plane through C and with the same height CD, such that the body will fall down it in a time B.

Proposition 23: A body falls from rest a vertical distance AC from rest at A, in a time AC. Let R be a distance more than twice and less than three times AC. It is required to find that plane CD on which the body, after falling through AC, will traverse a distance R in the time AC. (The plane becomes vertical when R is three times AC.)

We may remark that a velocity, once imparted to a body, will be perpetually maintained as long as the external causes of acceleration or retardation are removed. This condition is found only on horizontal planes; for downward-sloping planes there is an acceleration, and upward-sloping planes have a retardation. If a body, after descending down a plane, is deflected to an upward-inclined plane, the velocity acquired during the fall is then subject to a retardation. We can think of this as the superposition of a uniform motion upwards at the acquired speed together with a downward accelerating motion, starting from rest. If, after the fall, the body continues with a horizontal motion, there is only a uniform motion, while if it continues down a plane, there is in addition to this, a new uniform acceleration from rest.

From this it clearly follows that if a body falls a certain height down an inclined plane, and then continues up another plane, it will rise by the same height.

Proposition 24: If a body falls freely along a vertical (or inclined) line AE in a time t and then have its motion reflected up along an inclined plane BE, the distance which it will traverse along this plane in a time t is greater than once but less than twice AE.

Proposition 25: If a body falls along an inclined plane of length A, then continues along a horizontal plane of length B, the ratio of the time of descent along A to the time along B is equal to $2A : B$.

Proposition 26: A body falls freely along a vertical line AE in a time t; it is required to find an inclined plane BE such that when the body reaches E, its motion is reflected up BE, and traverses a given distance R, which is greater than once but less than twice AE, in a time t.

Proposition 27: A body descends along an inclined plane, AB, in a time t; it also falls another inclined plane, AC, longer than AB but of the same vertical height. The distance DC which it traverses on AC during a time t, is equal to $AB + AB\left(\frac{AC-AB}{AC}\right)$.

Proposition 28: Let AB be the vertical diameter of a circle, and let AE and EB be chords on this circle. Furthermore let F be a point on EB such that AF bisects the angle BAE; then the ratio of the time of fall down AB to the time of descent along AE followed by EB is equal to the ratio $AE : AE + EF$.

Proposition 29: A body falls a height AB, then moves along a horizontal line of fixed length BD. The least time it can take is when AB is half BD.

Proposition 30: Given a horizontal line BC and a vertical line BD, that inclined plane from C, meeting BD at E, down which a body will take the least time to fall, is such that BE equals BC.

Proposition 31: More generally, if the line BD is inclined, the least time of fall occurs when CE bisects the angle PCQ where CQ is vertical, and CP is perpendicular to BD.

Proposition 32: Moreover, BC is equal to BE; and if CP and CQ are two other lines such that CE bisects the angle PCQ, the times of descent down CP and CQ are the same.

Proposition 33: A body falls a height AB from rest in a time t. It is required to find the initial speed of the body in order to fall down an inclined plane AC with height AB, in the same time t.

Proposition 34: A body falls an inclined plane AB from rest in a time t. It is required to find the height CA above A, such that the body falls down it and then along AB in the same time t.

Proposition 35: A body falls a height AB from rest, then continues along an inclined plane BC, reaching C in a time t. It is required to find how far it falls down the plane from rest in the same time t.

Proposition 36: Let AB be a chord on a vertical circle, with B being the lowest point of the circle, and the arc AB being less than a quadrant. AC and CB are two other chords with C on the arc AB. The time of descent down ACB, starting from rest, is less than along AB, and less also, by the same amount, than along CB.

It follows that the path of quickest descent from one point to another is not the shortest path joining them (i.e., a straight line), but the arc of a circle.

Proposition 37: A body falls from rest a vertical height AB in a time t. It is required to find points DE on an inclined plane AC, such that DE is equal to AB, and a body descending from rest at A down the plane traverses DE in the same time t.

Proposition 38: A body falls a vertical height AB from rest in a time P; it is then deflected into a horizontal direction and moves an amount X in the same time P. Another body falls a height AC from rest, and as before, is deflected into a horizontal plane to move a distance Y. It is required to find AB in terms of BC, if the ratio $X : Y$ is known.

Sagr.: This is truly a new science, based upon a single principle, that has escaped the attention of Archimedes, Apollonius and Euclid. I am sure that this topic will be taken up by speculative minds.

2.4 Fourth Day

Salv.: Today I propose to study projectiles, which are a combination of two motions, one uniform and horizontal, the other naturally accelerated downwards.

If a particle is given a speed on a horizontal plane, it will continue its motion perpetually, until it reaches the edge of the plane, when in addition it starts to accelerate downwards.

Proposition 1: A projectile describes a path which is a semi-parabola.

Simp.: All I know about parabolas is that they are curved lines treated by Apollonius.

Salv.: I will only need two properties, proved by Apollonius: given a parabola with a vertical axis AE, and a point B on the parabola, the square of the horizontal displacement BC from the axis is proportional to the distance AC down the axis; secondly, if CA is extended beyond the vertex to the point D such that AD is equal to AC, then BD is tangent to the parabola at B.

To prove the proposition, suppose a body moves uniformly along a horizontal plane AF, and at A it also acquires a downwards acceleration due to its weight. So while it continues its horizontal uniform motion, with the horizontal displacement proportional to time, it also falls vertical distances proportional to the square of time. If one considers the parabola whose axis is the vertical line through A, and passes through one point on the body's trajectory, it follows from the first property that the whole paths agree.

Sagr.: You are assuming that the horizontal does not alter the vertical motion in any way. But this can't be, since it is clear that the path of the projectile ought to pass through Earth's center. Moreover the horizontal plane goes uphill away from the axis, so that the motion along it is retarded. Not to mention the effect of the air or water resistance.

Salv.: All of these are real difficulties; I cannot but grant that the proposition holds only approximately. But the horizontal can be considered a plane for distances that are small compared to the Earth's radius. After all, the greatest range of an artillery piece about 4 miles, compared to the 4000 miles to Earth's center.

The resistance caused by the medium is a more serious objection; it affects both the horizontal motion as well as the vertical one. No longer are they uniform, for the resistance grows with the speed, as we had discussed previously. Moreover it depends on the shape and density of the projectile body itself. We may only suppose that the shape is such as to offer the least resistance, such as cannon balls or arrows, made of dense material, and that the motion occurs in air.

Nevertheless, results from actual experiments agree with this theory extremely well. For example, if two balls, one of lead weighing ten times the other of oak, fall a height of, say, 200 cubits, they arrive at practically the same time, with one hardly two cubits below the other. This means that their final speed is nearly identical, about 400 cubits per unit time, where the unit time is the time of descent, about as high as that achieved by arrows. So the air resistance can be safely neglected in these instances.

Secondly, take two pendulums of length about five yards, with leaden balls; let one swing from about 80 deg, the other only 5 deg; the ratio of their speeds is thus about 16. If the air offers substantially more resistance to the fast ball than to the slower, there ought to be a difference in the periods; yet none is observed, even after counting hundreds of oscillations, and even though they do slow down. In fact, their swings reduce by the same proportion, which means that the air resistance is proportional to the speed of the ball.

With fired balls, the speed is much higher, even higher than the natural uniform speed acquired by the same ball after falling about a thousand cubits. I am convinced that a ball fired from a musket one cubit above ground would be flattened more than one fired directly down from a height of a hundred cubits.

Proposition 2: When the motion of a body consists of uniform horizontal and vertical motions, the square of the resultant momentum is equal to the sum of the squares of the two component momenta.

The reason is that for uniform motion, the horizontal and vertical speeds are proportional to the horizontal and vertical displacements, while the resultant speed is proportional to the diagonal.

We now wish to determine the speed of a projectile at any point on its trajectory. To do this we need a common standard to measure both the horizontal and vertical speeds. Although we have a clear standard for time: hours, minutes and seconds, we don't have one for velocity. I propose to convert a horizontal speed into that height which a body falling freely down it would acquire the said speed, so that all would agree about its size; this height is here called the *sublimity*.

Proposition 3: The vertical speed increases linearly with time, or equivalently, as the square root of the height.

Proposition 4: To determine the momentum of a projectile at each point in its given parabolic path.

The momentum is the diagonal of that triangle whose sides are the the vertical and horizontal momenta. Suppose the initial horizontal velocity has a sublimity AS, proportional to its square; this speed is maintained throughout the motion. Since the height AC is proportional to the square of the vertical speed, it follows that the resultant speed is, as a length, the sum of the lengths AS and AC, that is, the sublimity and the height.

Take note that the force or effect of the projectile does not depend only on its momentum but also on the nature of the target, in particular on the excess speed of the projectile over the target. A spear which reaches a man when their velocities are the same will harmlessly touch him. It also depends on what material it is made of, as well as the angle of the blow.

Sagr.: This brings to mind the following question: how come a 10 pound hammer can yield so much destruction on a target, when it does not yield to a much heavier body lying on it? How does one measure the force of a blow?

Salv.: For a long while I groped in the dark studying this problem, but we don't have the time to discuss my own ideas on this.

Proposition 5: To find that initial horizontal speed (or sublimity) for a projectile to describe a given parabola. (Using the property of parabolas, it is enough to take the tangent to any point on it.)

Proposition 6: The horizontal displacement of a projectile is twice the geometric mean of the altitude and the sublimity.

Proposition 7: Of all parabolic projectiles reaching the same horizontal displacement, the one with the least momentum is the one whose whose altitude is half this displacement.

Corollary: It follows that the maximum range occurs when a shot is fired at 45°, or the 6th point of a gunner's quadrant.

Proposition 8: The horizontal displacements of two parabolas described by projectiles fired with the same speed, but at angles of elevation which exceed and fall short of 45° by equal amounts, are equal.

The reason is that if one considers the triangles of momenta, the bigger one is similar to the smaller.

Proposition 9: If the sublimity is inversely proportional to the altitude, the horizontal displacement remains constant.

Proposition 10: The final momentum acquired by a projectile is equal to that which it would acquire in falling through a vertical distance equal to the sum of the sublimity and the altitude of the parabola.

Corollary: If the sum of the sublimity and the altitude remains constant, the final momenta are equal.

Proposition 11: Given the horizontal displacement and the final speed of a projectile, to find the altitude of the parabola.

Proposition 12: To compute and tabulate the ranges of all parabolas described by projectiles fired with the same initial speed.

Given an initial speed, there is a maximum height that can be reached, at 90° elevation; let it be 10000[17], which is the value of the tangent at 45°; its range is twice this value. From the point C where the projectile is fired, mark BC equal to 10000. Given an angle of elevation, say 50° (which can be assumed greater than 45° by proposition 8), find its tangent BE from the tables (e.g. 11918); take half of this BF (5959) and add to it the quantity FO, which

[17] Decimals were still not in wide use

is a third proportional to BF and the half of BC (e.g. $4195 = 5000^2/5959$); the desired horizontal displacement CR is then found from the following proportion $OB : BC = BC : CR$ (e.g. $10154 : 10000 = 10000 : CR$, so $CR = 9848$). The ranges are twice the horizontal displacements.

I have done this and obtained the following table of horizontal displacements and altitudes for projectiles with a constant initial speed:

67	7193			23	1527			68	8597		
66	7431	68	6947	22	1403	24	1654	67	8473	69	8716
65	7660	69	6691	21	1284	25	1786	66	8346	70	8830
64	7880	70	6428	20	1170	26	1922	65	8214	71	8940
63	8090	71	6157	19	1060	27	2061	64	8078	72	9045
62	8290	72	5878	18	955	28	2204	63	7939	73	9144
61	8480	73	5592	17	855	29	2351	62	7796	74	9240
60	8660	74	5300	16	760	30	2499	61	7650	75	9330
59	8829	75	5000	15	670	31	2653	60	7500	76	9415
58	8988	76	4694	14	585	32	2810	59	7347	77	9493
57	9135	77	4383	13	506	33	2967	58	7192	78	9567
56	9272	78	4067	12	432	34	3128	57	7034	79	9636
55	9397	79	3746	11	364	35	3289	56	6873	80	9698
54	9511	80	3420	10	302	36	3456	55	6710	81	9755
53	9613	81	3090	9	245	37	3621	54	6545	82	9806
52	9703	82	2756	8	194	38	3793	53	6378	83	9851
51	9781	83	2419	7	149	39	3962	52	6210	84	9890
50	9848	84	2079	6	109	40	4132	51	6040	85	9924
49	9903	85	1736	5	76	41	4302	50	5868	86	9951
48	9945	86	1391	4	49	42	4477	49	5696	87	9972
47	9976	87	1044	3	27	43	4654	48	5523	88	9987
46	9994	88	698	2	12	44	4827	47	5349	89	9998
45	10000	89	349	1	3	45	5000	46	5174	90	10000

Proposition 13: From the range of a parabola, to find its altitude.

I have another table with the altitudes and sublimities, but constant range. It is useful to estimate how faster the projectile needs to be thrown if it is to reach the same target at a different angle (the total momentum is the sum of the altitude and the sublimity). It is to be noted that if the sublimity and altitude of an angle, say 40 deg, are 4196 and 5959, that of 50 deg are the same values inverted, 5959 and 4196.

23	2122	11779				68	12375	2020			
22	2020	12375	24	2226	11230	67	11779	2122	69	13025	1919
21	1919	13025	25	2332	10723	66	11230	2226	70	13737	1820
20	1820	13737	26	2439	10252	65	10723	2332	71	14521	1722
19	1722	14521	27	2548	9813	64	10252	2439	72	15388	1625
18	1625	15388	28	2659	9404	63	9813	2548	73	16354	1529
17	1529	16354	29	2772	9020	62	9404	2659	74	17437	1434
16	1434	17437	30	2887	8660	61	9020	2772	75	18660	1340
15	1340	18660	31	3004	8321	60	8660	2887	76	20054	1247
14	1247	20054	32	3124	8002	59	8321	3004	77	21657	1154
13	1154	21657	33	3247	7699	58	8002	3124	78	23523	1063
12	1063	23523	34	3373	7413	57	7699	3247	79	25723	972
11	972	25723	35	3501	7141	56	7413	3373	80	28356	882
10	882	28356	36	3633	6882	55	7141	3501	81	31569	792
9	792	31569	37	3768	6635	54	6882	3633	82	35577	703
8	703	35577	38	3906	6400	53	6635	3768	83	40721	614
7	614	40721	39	4049	6174	52	6400	3906	84	47572	525
6	525	47572	40	4196	5959	51	6174	4049	85	57150	437
5	437	57150	41	4346	5752	50	5959	4196	86	71503	349
4	349	71503	42	4502	5553	49	5752	4346	87	95405	262
3	262	95405	43	4662	5362	48	5553	4502	88	143181	174
2	174	143181	44	4828	5177	47	5362	4662	89	286499	87
1	87	286499	45	5000	5000	46	5177	4828	90	infinita	

Proposition 14: To find for each degree of elevation the altitudes and sublimities of parabolas of constant range.

Sagr.: This makes sense because if a ball is fired directly up, no matter with what force, it will not move horizontally at all. But I find it hard to believe that the same is true when it fired point blank, i.e., at zero degrees elevation.

Salv.: It is quite similar to a rope. The stretching force pulls it horizontally, while its weight pulls it down; no matter how much you stretch the rope, it can never be made perfectly straight. In fact the analogy is quite adept, for a rope takes on a shape which is very close to an inverted parabola. You can try it by drawing a parabola, and then hang a chain from your hands: when slightly stretched the approximation is a very good one.