Introduction to Logic
CSM1070

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What is Logic?

- Informally, Logic is a tool that allows us to derive a legitimate conclusion(s) from a set of given premises.
- Logic is described in a language called **First Order Logic (FOL)**.
- We need to use such notation because:
  - English is ambiguous.
  - Convention and Clarity are crucial.

Atomic Sentences

- You can think of **Atomic Sentences** as the most basic of English sentences consisting of names connected by **predicates**:
  - Andrew ran.
  - Andrew saw Mark.
- Individual Constants:
  - These are symbols used to refer to some fixed individual **object** e.g. Andrew.
  - The FOL equivalent of names.

Individual Constants

- Individual Constants:
  - However in FOL these names are required to refer to **exactly one** object. In English it may be possible state that **Andrew saw Andrew**, where the two Andrews are different people. In FOL this is unacceptable.
  - An object may have more than one name. For example Andrew and Andy may refer to the same person (think of a unique nickname).

Predicate Symbols (1)

- Predicate symbols are used to denote:
  - A property of an object.
  - A relation between one or more objects.
- In Andrew saw Mark,
  - **saw** is the predicate.
  - Andrew, and Mark become the **arguments** of the predicate.
  - In this case the predicate saw is used as a **binary predicate**, since it takes two arguments.

Predicate Symbols (2)

- In English some predicates may be optional:
  - Andrew saw
  - Andrew saw Mark
- In FOL however, each predicate has a fixed number of arguments. This number is called the **arity** (from un**ary**, bin**ary**, etc…).
- Examples:
  - **AtHome(**Andrew**)**
  - **Taller(**Andrew,Mark**)**
  - **Between(**6,5,7**)**

Predicates Symbols (3)

- In English, predicates are sometimes vague. For example, **Claire** (who is six years old) **is young**. She will not be young when she's 96 but there is no definite point where she stops being so. In contrast, in FOL predicates are interpreted by a **determinate property or relation**. (A definite fact).

Atomic Sentences

- If we wish to say that a is equal to be, we write **a = b**. The '=' predicate is written in **infix** notation (i.e. it lies between the arguments).
- The predicate Taller in Taller(Andrew,Mark) is written in **prefix** because it precedes the arguments.
- The order of arguments is important Taller(Andrew,Mark) is not the same as Taller(Mark,Andrew).

Propositions and Arguments (1)

- In FOL and Atomic Sentence are used to express **propositions** (claims about something).
- These claims are either **TRUE** or **FALSE**.
- Central in Logic, is the notion of a **argument** (not the same ‘argument’ seen previously).
- An argument is also called an **inference**.
- This is a sequence of propositions (**premises**) used to derive a **conclusion**.

Propositions and Arguments (2)

If I’m outside and it’s raining then I’ll get wet

Premises: I’m outside
It is raining

Conclusion: Therefore, I get wet

Validity of Arguments (1)

1. All men are mortal. Socrates is a man. Therefore, Socrates is mortal.
2. Larry is mortal and all men are mortal. So Larry is a man.

- The first is **logically valid** since its conclusion is a **logical consequence** of its premises. It is impossible for this conclusion to be false if the premises are true.
- What if Larry is a goldfish???

Validity of Arguments (2)

- Informally, we can say that:

  An argument is valid if and only if the conclusion must be true on the assumption that the premises are true.
- Note: this does not necessarily that the arguments must be true for the argument to be valid (e.g. if Socrates is a robot). In this case the conclusion is not guaranteed to be true.
- So… IF the premises are true AND the argument is valid THEN the conclusion is true.

Validity of Arguments (3)

Hamlet is red in the face
Hamlet is holding an empty bottle
Hamlet is swearing
Hamlet just fell over

Therefore, Hamlet is drunk

- Even though, the premises may be true, the argument may still be invalid if Hamlet was acting.

Validity of Arguments (4)

The SLT is in University
The University is in Malta

Therefore, the SLT is in Malta

- This is a good argument. If we accept the premises then we also accept the conclusion.

Compound Statements (1)

- Remember, sentences like “Andrew saw Mark” are atomic because they cannot be decomposed into smaller ones.
- Longer sentences are built by combining atomic sentences into compound ones:

  Andrew saw Mark <u>and</u> Mark was Swimming
- Atomic Sentences:
  - saw(Andrew,Mark)
  - was(Mark,Swimming)

Compound Statements (2)

- Compound sentence:
  - saw(Andrew,Mark) AND was(Mark,Swimming)

This is the SLT (atomic)

This is <u>not</u> the SLT (compound)

- Atomic Sentence:
  - is(This,SLT)
- Compound Statement:
  - NOT is(This,SLT)

Connectives

- The connectives AND, NOT, OR, etc… are described in FOL as follows:

English Version	FOL Name	Symbol
Not	Negation	¬
And	Conjunction	∧
Or	Disjunction	∨
If… Then…	Implication	→
…is equivalent to… …if and only if… (Andrew is alive iff his heart is beating)	Biconditional	↔

Negation (NOT)

- Consider, **strong(Peter)** and ¬**strong(Peter)**.
- Negation is a unary operator – it takes one argument.
- Truth table:

p	¬p
T	F
F	T

Conjunction (AND)

- **strong(Peter)** $\wedge$ **tall(Peter)**.
- Conjunction is a binary, infix operator – it takes two arguments.
- If **p** and **q** are propositions, the statement **p** $\wedge$ **q** is true iff both **p** and **q** are true.

p	**q**	**p** $\wedge$ **q**
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction (OR)

- **strong(Peter)** $\vee$ **tall(Peter)**.
- Disjunction is a binary, infix operator – it takes two arguments.
- If **p** and **q** are propositions, the statement **p** $\vee$ **q** is true iff either **p** or **q** are true.

p	**q**	**p** $\vee$ **q**
T	T	T
T	F	T
F	T	T
F	F	F

Implication (If… Then…)

- **strong(Peter) → tall(Peter)**.
- Implication is a binary, infix operator – it takes two arguments.
- If **p** and **q** are propositions, the statement **p → q** is true iff either **p** if false or **q** is true (or both).
- **p** is called the **antecedent** of the implication whilst **q** is called the **consequent**.
- An implication is sometimes called a **conditional**.

Tip: → is only false when the antecedent is true but the consequent is not.

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional (iff)

- 'An implication that works both ways'.
- If **p** and **q** are propositions, the statement **p ↔ q** is true iff **p** and **q** share the same truth value.

p	q	p ↔ q
T	T	T
T	F	F
F	T	F
F	F	T