Variables

- So far we have discussed the use of constants in relation to a predicate. For example, we wrote:

  IsHome(John)

  IsTaller(John,Mark)
- Now we introduce the notion of a **variable**.
- Consider the following expression using a variable:

  IsTaller(John,y)
- In FOL there can be infinitely many variables.
- By convention the letters u, v, w, x, y, and z are used.

Atomic Well-Formed Formulas

- **Atomic WFFs** are like the normal sentences we have seen, but make use of one or more variables in place of constants.
- So the following are Atomic WFFs:
  - IsHome(x)
  - IsTaller(John,z)
  - IsTaller(x,y)

Quantifiers

- An atomic WFF is not regarded as a sentence.
- However they can be converted into sentences by applying **quantifiers**.
- The quantifiers considered here are:
  - $\exists$ **Existential Quantifier "There exists"**
  - $\forall$ **Universal Quantifier "For all"**
- These allow us to specify that **all objects** or **at least one object** satisfies a condition.
- "There does not exist" may be written as $\neg\exists$ or $\nexists$

Universal Quantifier

- The universal quantifier is used to express claims like:

Everyone is at home	$\forall x$ AtHome(x)
Every lecturer is cool	$\forall x$ (Lecturer(x)$\rightarrow$Cool(x))

Existential Quantifier

- The existential quantifier is used to express claims like:

Someone is at home	$\exists x$ AtHome(x)
At least a lecturer is cool	$\exists x$ (Lecturer(x) $\wedge$ Cool(x))

Why do we need Quantifiers?

- One way of expressing the universal quantification $\forall$**x P(x)** as:

  $\mathbf{P(n_1) \wedge P(n_2) \wedge P(n_3) \wedge \ldots}$
- Similarly we can express the existential quantification of $\exists$**x P(x)** as:

  $\mathbf{P(n_1) \vee P(n_2) \vee P(n_3) \vee \ldots}$
- This becomes impractical. Say we are considering **x** to be:
  - Grains of sand (this is impractical)
  - The real numbers (this is impossible)
- Informally the universe of discourse is the “set of individual objects we are discussing now”.

DeMorgan's Laws for Quantifiers

- $\neg\forall \mathbf{x}\ \mathbf{P(x)} \equiv \exists \mathbf{x}\ \neg \mathbf{P(x)}$

  To see why:

  $\neg \mathbf{P(n_1)} \vee \neg \mathbf{P(n_2)} \vee \neg \mathbf{P(n_3)} \vee \ldots$

- $\neg\exists \mathbf{x}\ \mathbf{P(x)} \equiv \forall \mathbf{x}\ \neg \mathbf{P(x)}$

  To see why:

  $\neg \mathbf{P(n_1)} \wedge \neg \mathbf{P(n_2)} \wedge \neg \mathbf{P(n_3)} \wedge \ldots$