Disjunctions and Conjunctions

- A **Literal** is an atomic sentence or its negation (e.g. P, $\neg$P)
- A **disjunction** is a sequence of literals connected by $\vee$'s

  P $\vee$ Q $\vee$ R
- A **conjunction** is a sequence of literals connected by $\wedge$'s

  P $\wedge$ Q $\wedge$ R

Normal Forms

- A sentence is in **Negative Normal Form (NNF)** if it is built from literals using only $\vee$'s and $\wedge$'s.

  P $\wedge$ ($\neg$Q $\vee$ R)
- A sentence is in **Disjunctive Normal Form (DNF)** if it is a disjunction of one or more conjunctions (of literals)

  (P $\wedge$ Q) $\vee$ (R $\wedge$ S)
- A sentence is in **Conjunction Normal Form (CNF)** if it is a conjunction of one or more disjunctions (of literals)

  (P $\vee$ Q) $\wedge$ (R $\vee$ S)

Conversion between Normal Forms (1)

- Recall that in algebra it is useful to use distributive laws to convert expressions (no matter how complex) involving +'s and ×'s into ones that are sums of products. E.g.:

$$
\begin{aligned}
(a+b)(c+d) &= (a + b)c + (a+b)d \\
&= ac + bc + (a+b)d \\
&= ac + bc + ad + bd
\end{aligned}
$$

Conversion between Normal Forms (2)

- In exactly the same way, the distributive laws of logic (see previous slides) allow us to convert a sentence in NNF to DNF. E.g.:

$$
\begin{aligned}
(a \vee b) \wedge (c \vee d) &= (a \vee b) \wedge c \vee (a \vee b) \wedge d \\
&= a \wedge c \vee b \wedge c \vee (a \vee b) \wedge d \\
&= a \wedge c \vee b \wedge c \vee a \wedge d \vee b \wedge d
\end{aligned}
$$

Conversion between Normal Forms (3)

- Likewise we can use distribution to convert NNF to CNF. E.g.

$$
\begin{aligned}
(a \wedge b) \vee (c \wedge d) &= ((a \wedge b) \vee c) \wedge ((a \wedge b) \vee d) \\
&= (a \vee c) \wedge (b \vee c) \wedge ((a \wedge b) \vee d) \\
&= \\
&(a \vee c) \wedge (b \vee c) \wedge (a \vee d) \wedge (b \vee d)
\end{aligned}
$$

Conversion between Normal Forms (4)

- Sometimes DeMorgan's laws should be used to convert basic formulae (WFF) into NNF. E.g.

  $\neg(P \wedge Q) \equiv (\neg P \vee \neg Q)$
- Some sentences are WFF, NNF, DNF and CNF simultaneously:

  $(P \wedge Q \wedge R)$