Resolution - Preliminaries (1)

- Using many different inference rules such as modus ponens and modus tollens can be overwhelming.
- Instead we can use a general purpose inference rule called **resolution**.
- This makes automatic theorem provers such as PROLOG practical to develop.
- Before resolution can be applied, the sentence must be in normal form (i.e. uses only $\wedge$, $\vee$, and possibly $\neg$).
- Note that $P \rightarrow Q \equiv \neg P \vee Q$

Resolution - Preliminaries (2)

- A literal is an atomic formula or its negation.
- A clause is a set of literals, e.g. {A, B, $\neg$C}
- The symbol $\square$ denotes the empty clause {}.
- A clause **C** is satisfied by a truth assignment **h** if at least on literal in the clause is true.
- For example consider the truth assignment {T,F} on the clause {Plays(Kris), Sleeps(Kris)}.
- The first item in the truth assignment says that Plays(Kris) is T, so the clause is satisfied.
- In other words for a Clause **C** $\neq \square$, **h** satisfies **C** iff the disjunction of **C** is true.

Resolution - Preliminaries (3)

- Consider the CNF statement:
  (Play(Kris) $\vee$ Study(Kris)) $\wedge$ ($\neg$Study(Kris) $\vee$ Sleep(Kris))
- We have two clauses:
  - {Play(Kris), Study(Kris)}
  - {$\neg$Study(Kris), Sleep(Kris)}
- Clearly all the clauses must be satisfiable for the whole statement to be.
- CNF statements can be written down in set form:
  {{Play(Kris), Study(Kris)}, {$\neg$Study(Kris), Sleep(Kris)}}

The Resolution Inference Rule

$$\frac{A \vee B \qquad \neg B \vee C}{A \vee C}$$

$$\frac{\text{Play(Kris)} \vee \text{Study(Kris)} \qquad \neg\text{Study(Kris)} \vee \text{Sleep(Kris)}}{\text{Play(Kris)} \vee \text{Sleep(Kris)}}$$

- This is simple enough to understand:
  - It is obvious that either Study(Kris) or $\neg$Study(Kris), must be false.
  - So either Play(Kris) or Sleep(Kris) must be true.

Resolvents

- Let **C1** and **C2** be two clauses.
- **D** is a resolvent of **C1** and **C2** iff
  - A literal **L** exists in **C1**
  - **¬L** exists in **C2**
  - **D** is the merging of **{C1-L}** and **{C2-¬L}**
- For example:
  - **C1={A,¬B}, C2={A,B}, D = {A}**
  - **C1={A,B}, C2={¬A,¬B}, D={A,¬A} and {B,¬B}**

Resolution Proof by Refutation

- Suppose that the current knowledge base (KB) is converted into clause form and we are trying to prove a theorem:

  1) Add the negation of the theorem we are trying to prove in the set of clauses of the KB.

  2) Search for two resolvable clauses in the KB
    - If not found: THEOREM TO BE PROVEN IS FALSE
    - If found: resolve and add the result to the KB

  3) If a contradiction is reached then the theorem is true else goto **(2)**

Proving Modus Ponens

- In Modus Ponens we have:

$$\frac{\begin{array}{c} P \\ P \rightarrow Q \end{array}}{Q}$$

1. P is a unit clause {P}
2. Convert $P \rightarrow Q$ to $\neg P \vee Q$ to get $\{\neg P, Q\}$.
3. Add the negation of the theorem $\{\neg Q\}$
4. Resolvent of (1), (2) is {Q}
5. We have a contradiction {Q} and $\{\neg Q\}$
6. Proven.

Resolution Example (1)

Socrates is a man.
<u>All men are mortal.</u>
Socrates is a frog.

- Man(Socrates)
- Man(x) $\rightarrow$ Mortal(x)
- Frog(Socrates)
- Abbreviate:
  - M
  - $M \rightarrow T$
  - F

Resolution Example (2)

- Clauses:
  - $\{M\}$
  - $\{\neg M, T\}$ remember that $(M \rightarrow T) \equiv (\neg M \vee T)$
  - Add the negation of the conclusion: $\{\neg F\}$
- Apply resolution:
  - $\{M\} + \{\neg M, T\} = \{T\}$
- Nothing left to resolve and there is no contradiction in the conclusion – **The theorem is false.**