MEC 4011: Useful Equations

Ideal gas law $pv = RT$ or $pV = mRT$ or $pV = n\tilde{m}RT$, molar mass $\tilde{m} = \dfrac{\tilde{R}}{R}$, Universal gas constant $\tilde{R}$

Work $W = \int pdv$; and heat $Q = \int Tds$

For a cycle, the 1st law $\displaystyle\sum_{cycle} Q + \sum_{cycle} W = 0$

For Non-Flow, the 1st law $Q + W = u_2 - u_1$

Flow, the 1st law $\dot{m}(h_1 + \dfrac{c_1^2}{2} + z_1 g) + \dot{Q} + \dot{W} = \dot{m}(h_2 + \dfrac{c_2^2}{2} + z_2 g)$

Adiabatic mixing $\dot{m}_1(h_1 + \dfrac{c_1^2}{2}) + \dot{m}_2(h_2 + \dfrac{c_2^2}{2})\dot{Q} + \dot{W} = \dot{m}_3(h_3 + \dfrac{c_3^2}{2})$

Continuity equation $\dot{m} = cA\rho = \dfrac{cA}{v}$

$$s_2 - s_1 = c_p \ln\left[\dfrac{T_2}{T_1}\right] + R \ln\left[\dfrac{p_1}{p_2}\right]$$

Steady Flow Exergy $b = u + pv - T_o s = h - T_o s$

For a wet vapour, dryness fraction x, $u = u_f + xu_{fg}$; $h = h_f + xh_{fg}$; $s = s_f + xs_{fg}$

$c_p - c_v = R$ where specific heat at constant volume c_v ,specific heat at constant pressure c_p ,specific gas constant R

For a solid or liquid, no change of phase $dQ = mcdT$

$u = c_v T$ u is the specific internal energy

$h = c_p T$ h is the specific enthalpy

$h = u + pv$

Reversible Adiabatic process $pv^\gamma = C$, where $\gamma = \dfrac{c_p}{c_v}$

$$\dfrac{T_1}{T_2} = \left(\dfrac{v_2}{v_1}\right)^{n-1} \quad\text{and}\quad \dfrac{T_1}{T_2} = \left(\dfrac{p_1}{p_2}\right)^{(n-1)/n}$$

For a polytropic process $W = \dfrac{(p_2 v_2 - p_1 v_1)}{n-1} = \dfrac{R(T_2 - T_1)}{n-1}$, n is the polytropic index , $Q = \dfrac{(n-\gamma)}{(\gamma-1)}W$

For an isothermal process, the work done is $W = p_1 v_1 \ln\left(\dfrac{v_1}{v_2}\right)$

$$\eta_{cycle} = \dfrac{W}{Q_1} \; ; \; COP_{ref} = \dfrac{Q_2}{W} \; ; \; COP_{hp} = \dfrac{Q_1}{W} \; ; WorkRatio = \dfrac{NetWork}{GrossWork}$$

Carnot cycle $\quad \eta_{Carnot} = 1 - \dfrac{T_{low}}{T_{high}}$

$$s_2 - s_1 = c_p \ln\left[\dfrac{T_2}{T_1}\right] + R\ln\left[\dfrac{p_1}{p_2}\right]$$

$$c_v = \sum \dfrac{m_i}{m} c_{v_i} \text{ and } c_p = \sum \dfrac{m_i}{m} c_{pi} \text{ and } R = \sum \dfrac{m_i}{m} R_i$$

Ideal Otto Cycle $\quad \eta = 1 - \dfrac{1}{r_v^{\gamma-1}}$

ideal gas turbine cycle $\quad \eta = 1 - \dfrac{1}{r_p^{(\gamma-1)/\gamma}}$ and $WorkRatio = 1 - \dfrac{T_1}{T_3} r_p^{(\gamma-1)/\gamma}$

ideal Diesel cycle $\quad \eta = 1 - \dfrac{\beta^{\gamma} - 1}{(\beta-1)r_v^{\gamma-1}\gamma}$ where $\beta = \dfrac{v_3}{v_2}$ the cut-off artio

ideal mixed combustion/dual combustion diesel cycle $\quad \eta = 1 - \dfrac{k\beta^{\gamma} - 1}{[(k-1)+\gamma k(\beta-1)]r_v^{\gamma-1}}$ where $k = \dfrac{p_3}{p_2}$

Mean effective pressure $\quad MEP = \dfrac{W}{v_1 - v_2}$, where $v_1 - v_2$ is the swept volume

Specific Steam Consumption $\quad SCC = \dfrac{3600}{W} \dfrac{kg}{kWh}$ where $W = h_1 - h_2 \dfrac{kJ}{kg}$

Hydrogen H_2 2 $\qquad$ Oxygen O_2 32 $\qquad\qquad$ Nitrogen N_2 28 $\qquad$ Carbon C 12
Air 21% O_2 79% N_2

$$ExcessAir = \dfrac{actualAFR - StoichAFR}{StoichAFR} \times 100$$

Mixture Strength = Equivalence ratio $\quad \phi = \dfrac{AFR_s}{AFR_a}$

$$\dfrac{p_a}{p_T} = \dfrac{V_a}{V_T} = \dfrac{n_a}{n_T} \quad \textit{i.e.} \text{ Ratio of pressures =ratio of volumes = ratio of moles}$$

Simple Impulse

$$optimal\left(\dfrac{C_b}{C_{ai}}\right) = \dfrac{\cos\alpha_i}{2} \qquad\qquad \eta_{d\,max} = \cos^2\alpha_i \qquad Power_{max} = 2\dot{m}\,C_b^2$$

Two stage velocity compounded, Curtis Turbine

$$optimal\left(\frac{C_b}{C_{ai1}}\right) = \frac{\cos\alpha_{i1}}{4} \qquad \eta_{d\,max} = \cos^2\alpha_i \qquad Power_{max} = 8\dot{m}\,C_b^2$$

Impulse Reaction , Parsons type , 50% reaction

$$optimal\left(\frac{C_b}{C_{ai}}\right) = \cos\alpha_i \qquad \eta_{d\,max} = \frac{2\cos^2\alpha_i}{1+\cos^2\alpha_i} \qquad Power_{max} = \dot{m}\,C_b^2$$

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