1. The cylinder of a hydraulic ram is 254 mm internal diameter and 50 mm thick. Find the maximum hoop stress in the material and the radial and hoop stress at a point in the barrel 19 mm from the inner surface when the fluid pressure inside the cylinder is 9.24 MN/m$^2$.

   Ans: 28.83, 4.6, 24.2 MN/m$^2$

2. Determine the radial and hoop stresses at all points in a thick-walled cylinder of internal and external radii 76 mm and 152 mm respectively, when acted upon by internal and external pressures of 124 MN/m$^2$ and 46 MN/m$^2$ respectively. Show by means of a diagram how each stress varies over the thickness. For your distribution you may calculate the hoop and radial stresses at radii 76, 100, 125 and 152 mm.

   Ans: \( \begin{array}{c|c|c} r & \sigma_r & \sigma_\theta \\ \hline 76 & -124 & 84 \\ 152 & -46 & 6 \end{array} \)

3. A compound tube is made by shrinking one tube on another, the final dimensions being:-

   - External diameter: 254 mm
   - Internal diameter: 150 mm
   - Diameter at junction of tubes: 228 mm

   If the radial pressure on the external surface of the inner tube at the common 114 mm radius is 27.58 MN/m$^2$, determine the maximum tensile stress in the material of the outer tube.

   Ans: 257 MN/m$^2$

4. A mild steel sleeve 14 cm inside diameter and 20 cm outside diameter is subjected to an internal pressure of 45 N/mm$^2$. The sleeve is rigidly hooped so that the change in external diameter is zero. Determine the values of the radial and circumferential stresses at the inner and outer surfaces. Proceed from Lame’s equations and neglect axial stresses. \( \nu = 0.3 \)

   Ans: inner: -45 N/mm$^2$, +2.2 N/mm$^2$
   outer: -32.8 N/mm$^2$, -9.9 N/mm$^2$

5. Show that the maximum circumferential stress in a thick walled cylinder subjected to a uniformly distributed external pressure, the internal pressure being zero, is compressive and occurs at the bore. A thick cylinder 20 cm external and 10 cm internal diameter is subjected to a uniform pressure on its external surface only. Under the condition the change in bore volume is not to exceed 0.1% of the original. Obtain the value of the maximum circumferential stress in the cylinder and the permissible external pressure assuming the axial stress to be zero. Proceed from Lame’s equations.

   E = 207 kN/mm$^2$, \( \nu = 0.3 \)

   Ans: -121.6 N/mm$^2$, 45.6 N/mm$^2$

6. A bronze cylinder of 0.3 m internal diameter and 0.4 m external diameter is surrounded by a closely fitting steel sleeve of 0.45 m external diameter. Calculate the maximum hoop stresses in the (sleeve) steel and (cylinder) bronze when an internal pressure of
30 MN/m² is applied to the compound cylinder, assuming that before the application of this pressure, the contact stress at the common surface is zero.

\[ E_{\text{steel}} = 200 \text{ GN/m}^2 \quad \text{and} \quad \nu = 0.28 \]
\[ E_{\text{bronze}} = 100 \text{ GN/m}^2 \quad \text{and} \quad \nu = 0.35 \]

(Ans: \( p = 10.13 \text{ MN/m}^2 \)
\[ \sigma_c = +60.8 \text{ MN/m}^2 \text{ at inner surface of inner cylinder} \]
\[ \sigma_c = +86.4 \text{ MN/m}^2 \text{ at inner surface of outer cylinder} \]

7. A compound steel cylinder has a bore of 80 mm and an outside diameter of 160 mm and at the common surface a diameter of 120 mm. Find the radial pressure at the common surface which must be provided by shrinkage, if the resultant maximum hoop tension in the inner cylinder under a superimposed internal pressure of 60 MN/m² is to be half the value of the maximum hoop tension which would be produced in the inner cylinder if that cylinder alone were subjected to an internal pressure of 60 MN/m².

Determine the final hoop tensions at the inner and outer surfaces of both cylinders under the internal pressure of 60 MN/m², and sketch a graph to show how the hoop tension varies across the cylinder wall.

(Ans: 6.11 MN/m², 77.42 MN/m², 55.7 MN/m², 78 MN/m², 39.66 MN/m²)

8. A steel ring of outer diameter 0.3 m and internal diameter 0.2 m is shrunk onto a solid steel shaft. The interference is arranged such that the radial pressure between the mating surfaces will not fall below 30 MN/m² whilst the assembly rotates in service. If the maximum circumferential stress is limited to 240 MN/m², find the maximum speed at which the assembly can be rotated. It may be assumed that no relative slip occurs between the shaft and the ring.

For steel, \( E = 208 \text{ GN/m}^2, l = 7470 \text{ kg/m}^3, \nu = 0.3 \)

(Ans: \( \omega = 9882 \text{ R.P.M.} \))

9. A steel rotor disc which is part of a turbine assembly has a uniform thickness of 40 mm. The disc has an outer diameter of 0.6 m and a central hole of 0.1 m diameter, the disc being shrunk onto the shaft. If there are 200 blades each of mass 0.153 kg pitched evenly around the periphery of the disc at an effective radius of 320 mm find the rotational speed at which yielding of the disc first occurs if \( \sigma_{\text{max}} \) is limited to 500 MN/m². Assume that the shrinkage allowance is such that at the speed at which yield first occurs, the contact pressure between the mating surfaces is zero.

For steel \( \nu = 0.3, \rho = 7470 \text{ kg/m}^3 \)

(Ans: \( \omega = 7435 \text{ R.P.M.} \))

10. The cross-section of a turbine rotor disc is designed for uniform strength under rotational conditions. The disc is keyed to a 60 mm diameter shaft at which point its thickness is a maximum. It then tapers to a minimum thickness of 10 mm at the outer radius of 250 mm where the blades are attached. If the design uniform stress of the disc is 250 MN/m² at the design speed of 12000 r.p.m., what is the required maximum thickness.

(Ans: \( \text{t}_{\text{max}} = 42.77 \text{ mm} \))
11. A flat 0.5 m outer diameter, 0.1 m inner diameter and 0.075 m thick steel disk is shrunk onto a steel shaft. If the assembly is to run at speeds up to 6900 r.p.m. find
(a) the shrinkage allowance
(b) the maximum stress when not rotating
(c) the maximum stress when rotating

ρ = 7800 kg/m$^3$, E = 200 GPa, $\nu = 0.3$

For the problem assume that at the speed of 6900 rpm the contact pressure between the mating surfaces is zero.

(Ans: $5.26 \times 10^{-5}$ on radius, 109.2 MPa, 212 MPa)