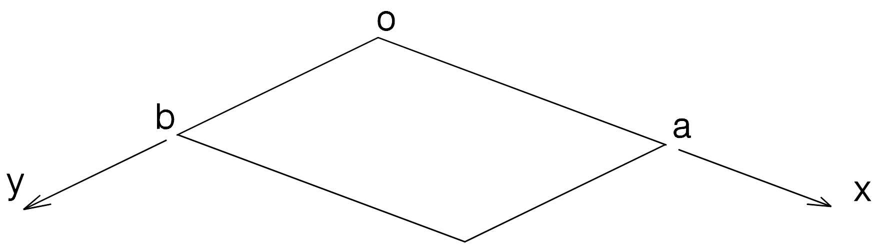
Tutorial 12 – Thin Rectangular and Circular plates

1. A rectangular plate measuring $a \times b$ is simply supported on all four sides. The plate is subjected to a rate of loading $P = P_o \ Sin\frac{\pi x}{a} \ Sin\frac{\pi y}{b}$.



Using an energy method find the transverse displacement w anywhere along the plate. The deflected form of the plate may be assumed to be $w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin\frac{m \pi x}{a} \sin\frac{n \pi y}{b}$.

$$\textbf{Ans:} \quad w = \frac{P_o}{D\pi^4\left(\frac{1}{a^2}+\frac{1}{b^2}\right)^2} \ Sin\frac{\pi x}{a} \ Sin\frac{\pi y}{b}$$

2. The simply supported rectangular plate shown in question (1) is subjected to a distributed load p given by

$$p = \frac{36P\,(a-x)(b-y)}{a^3\,b^3}.$$

Derive an expression for the deflection of the plate in terms of the constants *P*, *a*, *b*, and *D*. Use an energy method.

$$\textbf{Ans:} \quad w = \sum\sum A_{mn} \ Sin\frac{m\pi x}{a} \ Sin\frac{n\pi y}{b}$$

$$\text{where } A_{mn} = \frac{36P}{Dmn\pi^6\left\{\frac{m^4b^2}{4a^2}+\frac{m^2n^2}{2}+\frac{n^4a^2}{4b^2}\right\}}$$

3. A concentrated load *P* is applied at the centre of a simply supported rectangular plate (shown in problem 1). Determine, using an energy method, and if $m = n = 1$, $v = 0.3$, and $a = 2b$

(i) The maximum deflection.
(ii) The maximum stress in the plate.

Ans: (i) $w_{max} = 0.013\dfrac{Pb^2}{D}$ (ii) $\sigma_{y\,max} = 0.828\ \dfrac{P}{t^2}$

4. A uniformly loaded rectangular plate has its edges $y = 0$ and $y = b$ simply supported, the side $x = 0$ clamped, and the side $x = a$ is free. Assume a solution of the form $w = C\dfrac{x^2}{a^2}\ Sin\ \dfrac{\pi y}{b}$ where C is an undetermined coefficient. Apply the Ritz method to derive an expression for the deflected surface. Find the deflection at the middle of the free edge, if $a = b$ and $v = 0.3$.

Ans: w (middle point of free edge) = $0.0112\ \dfrac{p_o\, a^4}{D}$

5. A square plate of sides '*a*' has thickness '*t*'. The plate is simply supported on all edges and subjected to a uniform biaxial compression N. Determine the buckling stress by using an energy method.

Ans: $N_{CRITICAL} = \dfrac{2\pi^2 D}{a^2}$

6. A uniform rectangular flat plate is simply supported on three sides $x = 0$, $x = a$, $y = 0$ and is free to deflect laterally along the fourth side $y = b$. The plate is subjected to a uniform compressive edge loading N_x on two opposite simply supported edges. If the deformation of the plate can be represented by the series $w = \sum A_m\, y\ Sin\dfrac{m\pi x}{a}$ show that the critical value of N_x which will initiate instability is $N_{x\,CRITICAL} = \dfrac{\pi^2 D}{b^2}\left[\left(\dfrac{b}{a}\right)^2 + \dfrac{6(1-v)}{\pi^2}\right]$

7. A circular plate radius "*a*" having its edge clamped all round is loaded at the centre by a concentrated load P. Find equations for (a) the deflection (b) radial stress (c) circumferential stress.

Ans:

$$(a)\qquad \omega = \frac{P}{16\pi D}\left(2r^2 \ln\frac{r}{a} + a^2 - r^2\right)$$

$$(b)\qquad \sigma_r = \frac{3Pz}{\pi t^3}\left[(1+v)\ln\frac{a}{r} - 1\right]$$

$$(c)\qquad \sigma_\theta = \frac{3Pz}{\pi t^3}\left[(1+v)\ln\frac{a}{r} - v\right]$$

8. A circular clamped window of a seabed observation boat is subjected to a uniform pressure differential p_o per m^2 between the cabin and the outside. The plate is made of an isotopic material of tensile yield strength σ_{yp}, thickness 't', and raius 'a'. Use the shear strain energy theory $(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\,\sigma yp^2$ to predict the load carrying capacity of the plate.

(**Hint:** The circular plate first yields at the built-in edge)

Ans: $p_o = 1.5\,\dfrac{t^2}{a^2}\sigma_{yp}$

9. Determine the Bending Moments M_r and M_θ and the central deflection for a simply supported circular plate of radius "a" subjected to a uniformly distributed load p_o per m^2. Find also σ_r and σ_θ and the maximum value of σ if it occurs at the centre.

Ans:

$$w = \frac{p_o r^4}{64D} - \frac{p_o a^2 r^2}{32D}\frac{(3+v)}{(1+v)} + \frac{p_o a^4}{64D}\frac{(5+v)}{(1+v)}$$

$$M_r = \frac{p_o}{16}(3+v)(a^2 - r^2)$$

$$M_\theta = \frac{p_o}{16}\left[(3+v)a^2 - (1+3v)r^2\right]$$

$$\sigma_{r\max} = \sigma_{\vartheta\max} = \frac{3p_o a^2}{8t^2}(3+v)$$

10. A pressure control system includes a thin steel disk which is to close an electrical circuit by deflecting 1mm at the centre when the pressure attains a value of 3 MPa. Calculate the required disk thickness if it has a radius of 0.03 m and is built-in at the edge. $v = 0.3$ E = 200 GPa

Ans: $t = 1.275$ mm

11. A circular plate is simply supported round the outer boundary $r = a$. If the plate carries a point load P at the centre, derive the deflected shape of the plate and expressions for the radial and circumferential bending moments.

Ans:

$$w = \frac{P}{16\pi D}\left\{2r^2 \ln\frac{r}{a} + \frac{(3+v)}{(1+v)}\left(a^2 - r^2\right)\right\}$$

$$M_r = \frac{P}{4\pi}(1+v)\ln\frac{a}{r}$$

$$M_\theta = \frac{P}{4\pi}\left[(1+v)\ln\frac{a}{r} + 1 - v\right]$$

12. A circular flat plate is rigidly built-in along the outer boundary $r = a$. The plate has a rigid insert radius $r = b$ and carries a uniformly distributed load of intensity "p" per unit area. Obtain an expression for the deflected form of the plate and deduce the corresponding expression for a uniformly loaded continuous flat plate without a rigid insert.

Ans: $$w = \frac{p}{64D}\left[\left(r^4 - a^4\right) - 2\left(a^2 + b^2\right)\left(r^2 - a^2\right) + 4a^2b^2\ln\frac{r}{a}\right]$$