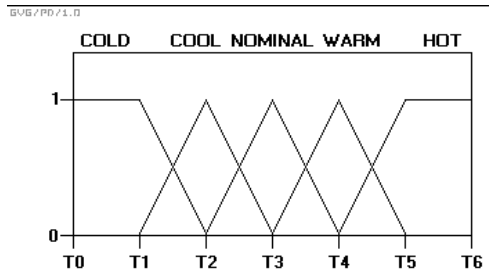
Introduction to Fuzzy Logic & Fuzzy Control

"Fuzzy logic" has become a common buzzword in machine control. However, the term itself inspires a certain skepticism, sounding equivalent to "half-baked logic" or "bogus logic". Some other nomenclature might have been preferable, but it's too late now, and fuzzy logic is actually very straightforward. Fuzzy logic is a way of interfacing inherently analog processes, that move through a continuous range of values, to a digital computer, that likes to see things as well-defined discrete numeric values.

For example, consider an antilock braking system, directed by a microcontroller chip. The microcontroller has to make decisions based on brake temperature, speed, and other variables in the system.

The variable "temperature" in this system can be divided into a range of "states", such as: "cold", "cool", "moderate", "warm", "hot", "very hot". Defining the bounds of these states is a bit tricky. An arbitrary threshold might be set to divide "warm" from "hot", but this would result in a discontinuous change when the input value passed over that threshold.

The way around this is to make the states "fuzzy", that is, allow them to change gradually from one state to the next. You could define the input temperature states using "membership functions" such as the following:



With this scheme, the input variable's state no longer jumps abruptly from one state to the next. Instead, as the temperature changes, it loses value in one membership function while gaining value in the next. At any one time, the "truth value" of the brake temperature will almost always be in some degree part of two membership functions: 0.6 nominal and 0.4 warm, or 0.7 nominal and 0.3 cool, and so on.

The input variables in a fuzzy control system are in general mapped into by sets of membership functions similar to this, known as "fuzzy sets". The process of converting a crisp input value to a fuzzy value is called "fuzzification".

A control system may also have various types of switch, or "ON-OFF", inputs along with its analog inputs, and such switch inputs of course will always have a truth value equal to either 1 or 0, but the scheme can deal with them as simplified fuzzy functions that are either one value or another.

Given "mappings" of input variables into membership functions and truth values, the microcontroller then makes decisions for what action to take based on a set of "rules", each of the form:

IF brake temperature IS warm AND speed IS not very fast

THEN brake pressure IS slightly decreased.

In this example, the two input variables are "brake temperature" and "speed" that have values defined as fuzzy sets. The output variable, "brake pressure", is also defined by a fuzzy set that can have values like "static", "slightly increased", "slightly decreased", and so on.

This rule by itself is very puzzling since it looks like it could be used without bothering with fuzzy logic, but remember the decision is based on a *set* of rules:

- All the rules that apply are invoked, using the membership functions and truth values obtained from the inputs, to determine the result of the rule.
- This result in turn will be mapped into a membership function and truth value controlling the output variable.
- These results are combined to give a specific ("crisp") answer, the actual brake pressure, a procedure known as "defuzzification".

This combination of fuzzy operations and rule-based "inference" describes a "fuzzy expert system".

Traditional control systems are based on mathematical models in which the the control system is described using one or more differential equations that define the system response to its inputs. Such systems are often implemented as "proportional-integral-derivative (PID)" controllers. They are the products of decades of development and theoretical analysis, and are highly effective.

If PID and other traditional control systems are so well-developed, why bother with fuzzy control? It has some advantages. In many cases, the mathematical model of the control process may not exist, or may be too "expensive" in terms of computer processing power and memory, and a system based on empirical rules may be more effective.

Furthermore, fuzzy logic is well suited to low-cost implementations based on cheap sensors, low-resolution analog-to-digital converters, and 4-bit or 8-bit one-chip microcontroller chips. Such systems can be easily upgraded by adding new rules to improve performance or add new features. In many cases, fuzzy control can be used to improve existing traditional controller systems by adding an extra layer of intelligence to the current control method.

Fuzzy Control in Detail

Fuzzy controllers are very simple conceptually. They consist of an input stage, a processing stage, and an output stage. The input stage maps sensor or other inputs, such as switches, thumbwheels, and so on, to the appropriate membership functions and truth values. The processing stage invokes each appropriate rule and generates a result for each, then combines the results of the rules. Finally, the output stage converts the combined result back into a specific control output value.

The most common shape of membership functions is triangular, although trapezoids and bell curves are also used, but the shape is generally less important than the number of curves and their placement. From three to seven curves are generally appropriate to cover the required range of an input value, or the "universe of discourse" in fuzzy jargon.

As discussed earlier, the processing stage is based on a collection of logic rules in the form of IF-THEN statements, where the IF part is called the "antecedent" and the THEN part is called the "consequent". Typical fuzzy control systems have dozens of rules.

Consider a rule for a thermostat:

IF (temperature is "cold") THEN (heater is "high")

This rule uses the truth value of the "temperature" input, which is some truth value of "cold", to generate a result in the fuzzy set for the "heater" output, which is some value of "high". This result is used with the results of other rules to finally generate the crisp composite output. Obviously, the greater the truth value of "cold", the higher the truth value of "high", though this does not necessarily mean that the output itself will be set to "high", since this is only one rule among many.

In some cases, the membership functions can be modified by "hedges" that are equivalent to adjectives. Common hedges include "about", "near", "close to", "approximately", "very", "slightly", "too", "extremely", and "somewhat". These operations may have precise definitions, though the definitions can vary considerably between different implementations. "Very", for one example, squares membership functions; since the membership values are always less than 1, this narrows the membership function. "Extremely" cubes the values to give greater narrowing, while "somewhat" broadens the function by taking the square root.

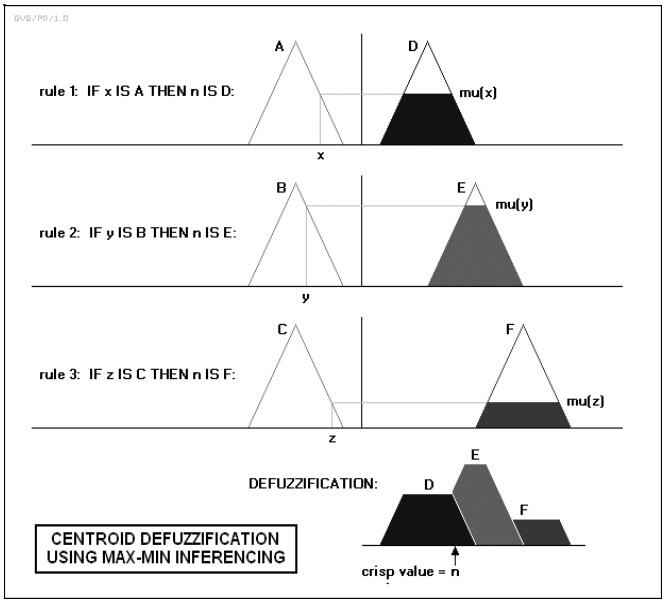
In practice, the fuzzy rule sets usually have several antecedents that are combined using fuzzy operators, such as AND, OR, and NOT, though again the definitions tend to vary: AND, in one popular definition, simply uses the minimum weight of all the antecedents, while OR uses the maximum value. There is also a NOT operator that subtracts a membership function from 1 to give the "complementary" function.

There are several different ways to define the result of a rule, but one of the most common and simplest is the "max-min" inference method, in which the output membership function is given the truth value generated by the premise.

Rules can be solved in parallel in hardware, or sequentially in software. The results of all the rules that have fired are "defuzzified" to a crisp value by one of of several methods. There are dozens in theory, each with various advantages and drawbacks.

The "centroid" method is very popular, in which the "center of mass" of the result provides the crisp value. Another approach is the "height" method, which takes the value of the biggest contributor. The centroid method favors the rule with the output of greatest area, while the height method obviously favors the rule with the greatest output value.

The example below demonstrates max-min inferencing and centroid defuzzification for a system with input variables "x", "y", and "z" and an output variable "n". Note that "mu" is standard fuzzy-logic nomenclature for "truth value":

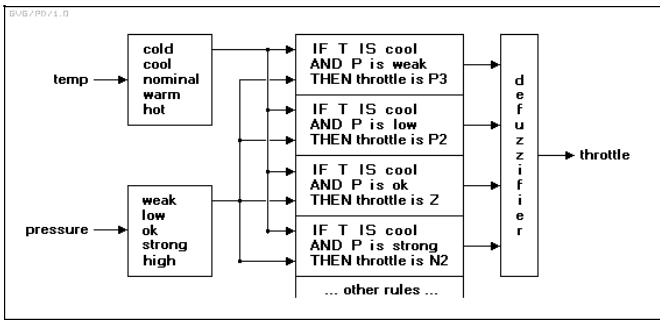


Notice how each rule provides a result as a truth value of a particular membership function for the output variable. In centroid defuzzification the values are OR'd, that is, the maximum value is used and values are not added, and the results are then combined using a centroid calculation.

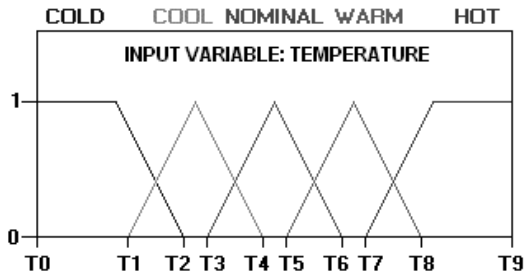
Fuzzy control system design is based on empirical methods, basically a methodical approach to trial-and-error. The general process is as follows:

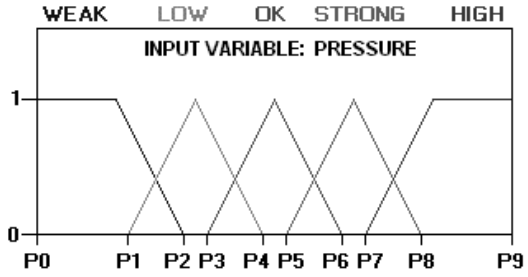
- Document the system's operational specifications and inputs and outputs.
- Document the fuzzy sets for the inputs.
- Document the rule set.
- Determine the defuzzification method.
- Run through test suite to validate system, adjust details as required.
- Complete document and release to production.

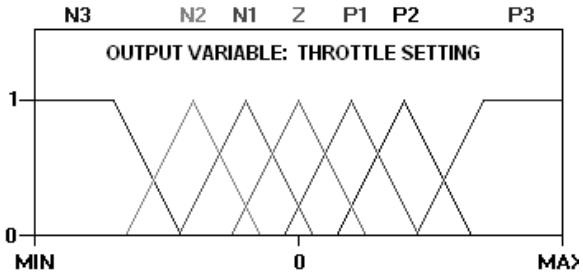
As a general example, consider the design of a fuzzy controller for a steam turbine. The block diagram of this control system appears as follows:



There are two input variables, temperature and pressure, and a single output variable, the turbine throttle setting. The turbine's operation can be reversed, so the throttle setting can be positive or negative. The fuzzy set mappings are shown below:







The throttle settings are defined as follows:

N3: Large negative.

N2: Medium negative.

N1: Small negative.

Z: Zero.

P1: Small positive.

P2: Medium positive.

P3: Large positive.

The rule set includes such rules as:

Rule 1: IF temperature IS cool AND pressure IS weak, THEN throttle is P3.

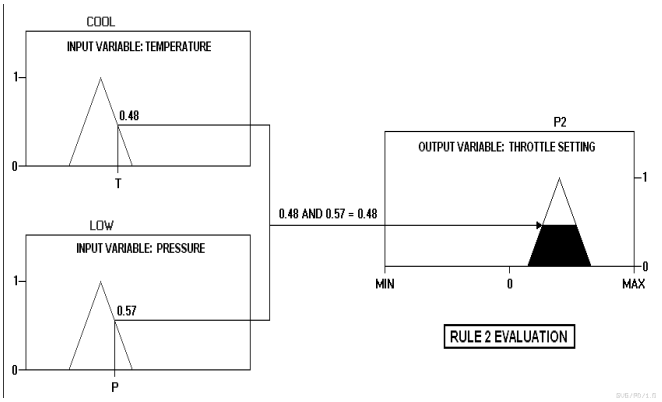
Rule 2: IF temperature IS cool AND pressure IS low, THEN throttle is P2.

rule 3: IF temperature IS cool AND pressure IS ok, THEN throttle is Z.

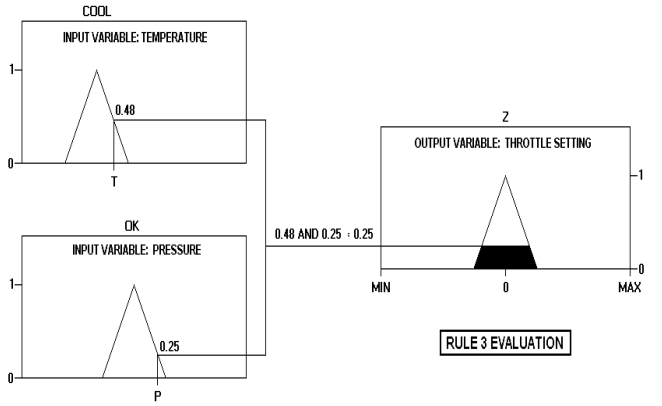
Rule 4: IF temperature IS cool AND pressure IS strong, THEN throttle is N2.

In practice, the controller accepts the inputs and maps them into their membership functions and truth values. These mappings are then fed into the rules. If the rule specifies an AND relationship between the mappings of the two input variables, as the examples above do, the minimum of the two is used as the combined truth value; if an OR is specified, the maximum is used. The appropriate output state is selected and assigned a membership value at the truth level of the premise. The truth values are then defuzzified.

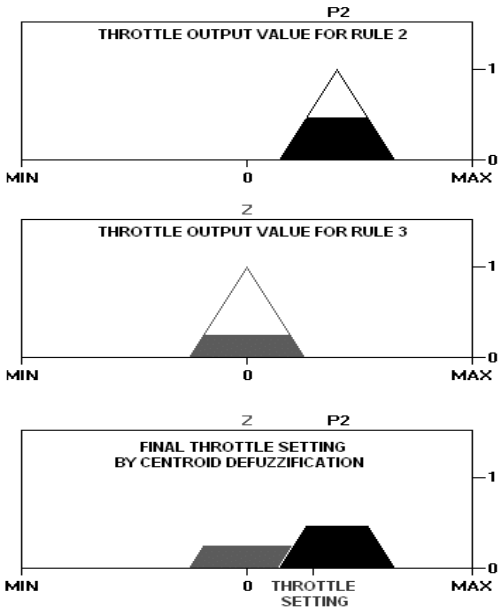
For an example, assume the temperature is in the "cool" state, and the pressure is in the "low" and "ok" states. The pressure values ensure that only rules 2 and 3 fire. Rule 2 is evaluated as follows:



Rule 3 is evaluated as follows:



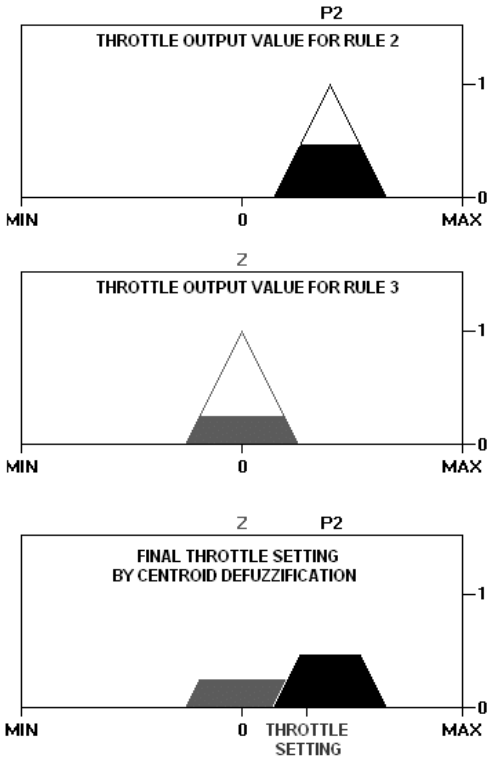
The two outputs are then combined:



The output value will adjust the throttle and then the control cycle will begin again to generate the next value.

Building a Fuzzy Controller

Consider implementing with a microcontroller chip a simple feedback controller:



A fuzzy set is defined for the input error variable “e”, and the derived change in error from the previous error input, “delta”, as well as the “output”, as follows:

LP: large positive

SP: small positive

ZE: zero

SN: small negative

LN: large negative

If the error ranges from -1 to +1, with the analog-to-digital converter used having a resolution of 0.25, then the input variable’s fuzzy set (which, in this case, also applies to the output variable) can be described very simply as a table, with the error / delta / output values in the top row and the truth values for each membership function arranged in rows beneath:

	-1	-0.75	-0.5	-0.25	0	0.25	0.5	0.75
mu(LP)	0	0	0	0	0	0	0.3	0.7
mu(SP)	0	0	0	0	0.3	0.7	1	0.7
mu(ZE)	0	0	0.3	0.7	1	0.7	0.3	0
mu(SN)	0.3	0.7	1	0.7	0.3	0	0	0
mu(LN)	1	0.7	0.3	0	0	0	0	0

This table is a little tricky to interpret. It simply defines each of the elements of the fuzzy set (“LP” through “LN”) in terms of the values (“mu”) that they have relative to the given values in the top row.

It is important to remember that for the error / delta variables, the top row values are used to generate the mu values, while for the output variable, the mu values are used to generate the top row values.

Suppose this fuzzy system has the following rule base:

rule 1: IF e = ZE AND delta = ZE THEN output = ZE

rule 2: IF e = ZE AND delta = SP THEN output = SN

rule 3: IF e = SN AND delta = SN THEN output = LP

rule 4: IF e = LP OR delta = LP THEN output = LN

These rules are typical for control applications in that the antecedents consist of the logical combination of the error and delta signals, while the consequent is a control command output.

These rules can also be a little tricky to interpret. For example, what rule 1 says is that if the error value is the ZE fuzzy set element and the delta, or change from last error, value is the ZE fuzzy set element, then the output is also the ZE fuzzy set element. What is *very* tricky is that plugging actual numeric values into this rule does *not* give, at least directly, an actual value for the output setting.

The value for the output setting is defined by rule 1 as “ZE”. Looking at the fuzzy value table shows that “ZE” has its maximum value corresponding to the value of “0” on the top row of the table, and so this rule *always* gives an output setting of 0. Similarly, rule 2 always gives an output setting of “SN” or

-0.5, rule 3 always gives an output setting of “LP” or 1, and rule 4 always gives an output setting of “LN” or -1.

The thing to remember is that the actual output is a *combination* of the outputs of these four rules, weighted by the values by the appropriate mu values provided by the rule calculation, using the centroid calculation:

SUM(I = 1 TO 4 OF (mu(I) * output(I))) / SUM(I = 1 TO 4 OF mu(I))

The result of this calculation gives the actual output setting.

Now, suppose that at a given time we have:

error = 0.25

delta = 0.5

Then the corresponding mu values can be obtained by simply taking them from the appropriate columns of the fuzzy set table:

	error = 0.25	delta = 0.5
mu(LP)	0	0.3
mu(SP)	0.7	1
mu(ZE)	0.7	0.3
mu(SN)	0	0
mu(LN)	0	0

These values can then be plugged into the rules to give output values. Taking the first rule:

rule 1: IF e = ZE AND delta = ZE THEN output = ZE

The weighting for the output, “mu(1)”, is produced with the simple calculation:

mu(1) = MIN(0.7, 0.3) = 0.3

As mentioned above, the output value of this rule is always 1:

output(1) = 0

The other rules give:

rule 2: IF e = ZE AND delta = SP THEN output = SN

mu(2) = MIN(0.7, 1) = 0.7

output(2) = -0.5

rule 3: IF e = SN AND delta = SN THEN output = LP

mu(3) = MIN(0.0, 0.0) = 0

output(3) = 1

rule 4: IF e = LP OR delta = LP THEN output = LN

mu(4) = MAX(0.0, 0.3) = 0.3

output(4) = -1

The centroid computation yields:

$$\frac{mu(1)*output(1) + mu(2)*output(2) + mu(3)*output(3) + mu(3)*output(3)}{mu(1) + mu(2) + mu(3) + mu(4)}$$

$$= \frac{(0.3 * 0) + (0.7 * -0.5) + (0 * 1) + (0.3 * -1)}{0.3 + 0.7 + 0 + 0.3}$$

$$= \frac{0 - 0.35 + 0 - 0.3}{1.3}$$

$$= -0.65 / 1.3$$

$$= -0.5$$

— for the final control output. . Of course the hard part is figuring out what rules actually work correctly in practice.

* If you have problems figuring out the centroid equation, remember that a centroid is defined by summing all the moments (location times mass) around the center of gravity and equating the sum to zero. So if X0 is the center of gravity, Xi is the location of each mass, and Mi is each mass, this gives:

0 = (X1 - X0) * M1 + (X2 - X0) * M2 + ... + (Xn - X0) * Mn

0 = (X1 * M1 + X2 * M2 + ... + Xn * Mn) - X0 * (M1 + M2 +..+ Mn)

X0 * (M1 + M2 + ... + Mn) = (X1 * M1 + X2 * M2 + ... + Xn * Mn)

$$X0 = \frac{(X1 * M1 + X2 * M2 + ... + Xn * Mn)}{(M1 + M2 + ... + Mn)}$$

In our example, the values of mu correspond to the masses, and the values of X to location of the masses.

History & Applications

Fuzzy logic was first proposed by Lotfi A. Zadeh of the University of California at Berkeley in a 1965 paper. He elaborated on his ideas in a 1973 paper that introduced the concept of “linguistic variables”, which in this article equates to a variable defined as a fuzzy set. Other research followed, with the first industrial application, a cement kiln built in Denmark, coming on line in 1975.

Fuzzy systems were largely ignored in the US because they were associated with artificial intelligence, a field that periodically oversells itself and which did so in a big way in the mid-1980s, resulting in a lack of credibility in the commercial domain.

The Japanese did not have this prejudice. Interest in fuzzy systems was sparked by Seiji Yasunobu and Soji Miyamoto of Hitachi, who in 1985 provided simulations that demonstrated the superiority of fuzzy control systems for the Sendai railway. Their ideas were adopted, and fuzzy systems were used to control accelerating, braking, and stopping when the line opened in 1987.

Another event in 1987 helped promote interest in fuzzy systems. During a international meeting of fuzzy researchers in Tokyo that year, Takeshi Yamakawa demonstrated the use of fuzzy control, through a set of simple dedicated fuzzy logic chips, in an “inverted pendulum” experiment. This is a classic control problem, in which a vehicle tries to keep a pole mounted on its top by a hinge upright by moving back and forth.

Observers were impressed with this demonstration, as well as later experiments by Yamakawa in which he mounted a wine glass containing water or even a live mouse to the top of the

pendulum. The system maintained stability in both cases. Yamakawa eventually went on to organize his own fuzzy-systems research lab to help exploit his patents in the field.

Following such demonstrations, the Japanese became infatuated with fuzzy systems, developing them for both industrial and consumer applications. In 1988 they established the Laboratory for International Fuzzy Engineering (LIFE), a cooperative arrangement between 48 companies to pursue fuzzy research. Japanese companies developed a wide range of products using fuzzy logic, ranging from washing machines to auto focus cameras and industrial air conditioners.

Some work was also performed on fuzzy logic systems in the US and Europe, and a number of products were developed using fuzzy logic controllers. However, little has been said about the technology in recent years, which implies that it has either become such an ordinary tool that it is no longer worth much comment, or it turned out to be an industrial fad that has now generally died out.

Fuzzy logic is used directly in very few applications. The Sony PalmTopapparently uses a fuzzy logic decision tree algorithm to performhandwritten (well, computer lightpen) Kanji character recognition.

Most applications of fuzzy logic use it as the underlying logic systemfor fuzzy expert systems

Comments, Sources, & Revision History

I have no background in control systems nor in fuzzy logic, but something about fuzzy logic intrigues me. I had accumulated notes on the subject off and on for a few years, and then reached a threshold where I decided to merge them into my own tutorial, focusing on control applications.

Sources for this lesson include:

- “Designing With Fuzzy Logic” by Kevin Self, IEEE SPECTRUM, November 1990, 42:44,105.
- “Fuzzy Fundamentals” by Earl Cox, IEEE SPECTRUM, October 1992, 58:61. Notes from this article constitute the core of this tutorial.
- “Fuzzy Logic Flowers In Japan” by Daniel G. Schwartz & George J. Klir, IEEE SPECTRUM, July 1992, 32:35.
- “Clear Thinking On Fuzzy Logic” by Lawrence A. Berardinis, MACHINE DESIGN, 23 April 1992, 46:52.
- “Fuzzy Controller Challenges 8-bit MCUs”, COMPUTER DESIGN, November 1995.
- “Dishwasher Cleans Up With Fuzzy Logic”, MACHINE DESIGN, 23 March 1995.
- “How To Design Fuzzy Logic Controllers”, MACHINE DESIGN, 26 November 1992.
- “A Case For Fuzzy Logic” by Byron Miller, EMBEDDED SYSTEMS PROGRAMMING, December 1995, 42:70.

Case Study: Fuzzy Traffic Light Controller

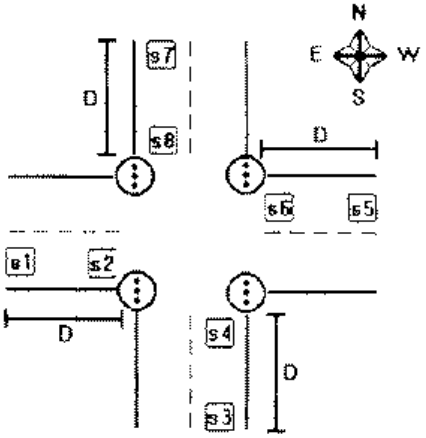
This part of the report describes the design procedures of a real life application of fuzzy logic: A Smart Traffic Light Controller. The controller is suppose to change the cycle time depending upon the densities of cars behind green and red lights and the current cycle time.

Background

In a conventional traffic light controller, the lights change at constant cycle time, which is clearly not the optimal solution. It would be more feasible to pass more cars at the green interval if there are fewer cars waiting behind the red lights. Obviously, a mathematical model for this decision is enormously difficult to find. However, with fuzzy logic, it is relatively much easier.

Fuzzy Design

First, eight incremental sensors are put in specific positions as seen in the diagram below.



The first sensor behind each traffic light counts the number cars coming to the intersection and the second counts the cars passing the traffic lights. The amount of cars between the traffic lights is determined by the difference of the reading of the two sensors. For example, the number of cars behind traffic light North is s7-s8.

The distance D, chosen to be 200ft., is used to determine the maximum density of cars allowed to wait in a very crowded situation. This is done by adding the number of cars between to paths and dividing it by the total distance. For instance, the number of cars between the East and West street is (s1-s2)+(s5-s6)/400.

Next comes the fuzzy descision process which uses the three

step mentioned above(fuzzyification, rule evaluation and defuzzification).

Step 1

As before, firstly the inputs and outputs of the design has to be determined. Assuming red light is shown to both North and South streets and distance D is constant, the inputs of the model consist of :

1. Cycle Time
2. Cars behind red light
3. Cars behind green light

The cars behind the light is the maximum number of cars in the two directions. The corresponding ouput parameter is the probabilty of change of the current cycle time. Once this is done, the input and output parameters are divided into overlapping member fuctions, each function corresponding to different levels. For inputs one and two the levels and their corresponding ranges are zero(0,1), low(0,7), medium(4,11), high(7,18), and chaos(14,20). For input 3 , the levels are very short(0,14), short(0,34), medium(14,60), long(33,88), very long(65,100), limit(85,100). The levels of output are no(0), probably no(0.25), maybe(0.5), probably yes (o.75), and yes(1.0). Note: For the output, one value(singleton position) is associated to each level instead of a range of values. The corresponding graphs for each of these membership function is drawn in the similar way above.

Step 2

The rules, as before are formulated using a series of if-then statements, combined with AND/OR operators. Ex: if cycle time is medium AND Cars Behind Red is low AND Cars Behind Green is medium, then change is Probably Not. With three inputs, each having 5,5,and 6 membership functions, there are a combination of 150 rules. However using the minimum or maximum criterion some rules are combined to a total of 86.

Step 3

This process, also mentioned above converts the fuzzy set output to real crisp value. The method used for this system is *center of gravity*:

Crisp Output={Sum(Membership Degree*Singleton Position)}/(Membership degree) For example, if the output membership degree, after rule evaluation are:
Change Probability Yes=0, Change Probability Probably Yes=0.6, Change Probability Maybe=0.9, Change Probability Probably No= 0.3, Change Probability No=0.1
then the crisp value will be: Crisp Output=(0.1*0.00) +(0.3*0.25)+(0.9*0.50)+(0.6*0.75)+(0*1.00)/ 0.1+0.3+0.9+0.6+0 =0.51

Is Fuzzy Controller better ?

Testing of the Controller

The fuzzy controller has been tested under seven different kinds of traffic conditions from very heavy traffic to very lean traffic. 35 random chosen car densities were grouped according to different periods of the day representing those traffic conditions.

Performance Evaluation

The performance of the controller was compared with that of a conventional controller and a human expert. The criteria used for comparison were number of cars allowed to pass at one time and average waiting time. A performance index which maximises the traffic flow and reduces the average waiting time was developed. A means of calculating the average waiting time was also developed, however, a detailed calculation of this evaluation is beyond the scope of this article. All three traffic controller types were compared and can be summarized with the following graph of performance index in all seven traffic categories.

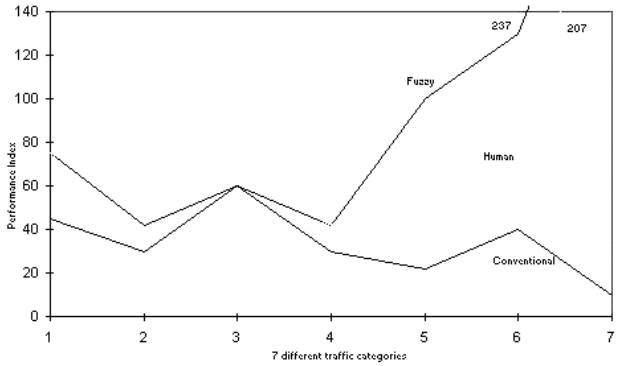


Fig: Performance Index for 7 different traffic catagories

Conclusion

The fuzzy controller passed through 31% more cars, with an average waiting time shorter by 5% than the theoretical minimum of the conventional controller. The performance also measure 72% higher. This was expected. However, in comparison with a human expert the fuzzy controller passed through 14% more cars with 14% shorter waiting time and 36% higher performance index. Result: Machine beats Man!!!!

In conclusion, as Man gets hungry in finding new ways of improving our way of life, new, smarter machines must be created. Fuzzy logic provides a simple and efficient way to meet these demands and the future of it is limitless.

Notes