CCE2301—MATLAB: Practical 4

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Differential Equations

Objective

The objective of this practical is to solve differential equations using MATLAB.

Procedure

A ship has a heading angle ψ. It is steered using a rudder which has an angle δ. The ship behaviour can be modelled with the differential equation:

$$\tau_1 \tau_2 \dddot{\psi} + (\tau_1 + \tau_2)\ddot{\psi} + \dot{\psi} = K\delta$$

The parameter values are

$$\tau_1 = 10$$
$$\tau_2 = 5$$
$$K = 2$$

Note that $\dot{\psi} = \frac{d\psi}{dt}$, $\ddot{\psi} = \frac{d^2\psi}{dt^2}$, and $\dddot{\psi} = \frac{d^3\psi}{dt^3}$.

1. Convert the differential equation into state-variable form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}\delta$$

   The state variable $\mathbf{x}$ is defined as

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \psi \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix}$$

   Write down the matrices $\mathbf{A}$ and $\mathbf{b}$.

2. The rudder angle δ is to be controlled as follows:

$$\delta = -\mathbf{k}^{\mathrm{T}}\mathbf{x} + g\psi_r$$

   The parameters are

$$\mathbf{k} = \begin{bmatrix} 0.3 \\ 3.5 \\ 10 \end{bmatrix}$$

$$g = 0.3$$

$$\psi_r = \begin{cases} 2 & \text{if } t < 50 \\ -2 & \text{if } t \geq 50 \end{cases}$$

   Write a function with the following definition:

```
function d = delta(t, x)
```

   This function should calculate δ for the given values of t and $\mathbf{x}$.

3. Write a function with the following definition:

```
function xdot = ship(t, x)
```

   This function should calculate the derivative of $\mathbf{x}$ with respect to t using the matrices $\mathbf{A}$ and $\mathbf{b}$, and using the function `delta`.

4. Solve the differential equation using `ode45` for $t \in [0, 100]$. Assume zero initial conditions.

5. On the same axis, plot ψ, $\dot{\psi}$ and $\ddot{\psi}$ against time t.

6. Plot the rudder angle δ against time t.

Report

Your report should include any MATLAB scripts and functions that you wrote, the plots, and any observations and comments. Note that printed plots should be labelled properly.