Simple Harmonic Motion
Introductory Worksheet

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Author: Russell Mizzi
russell.mizzi@gmail.com

Junior College Physics Department

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1. Define the term ‘simple harmonic motion’

2. Draw a graph of acceleration \(a\) against displacement \(y\) for a simple harmonic oscillator given that the amplitude of oscillation is \(A\). On your graph clearly indicate the maximum values of the acceleration \(a\) in terms of the angular frequency \(\omega\) and the amplitude \(A\).

3. A body oscillates horizontally with an amplitude of 50 mm and a frequency of 8 Hz. Calculate the acceleration

   (a) at the centre of the motion
   (b) at the extremities
   (c) at a position midway between the centre and the extremities

4. A mass \(m\) is attached to the lower end of a light vertical spring of stiffness \(k\) hanging from a fixed support. The resulting extension \(e\) when the system is in equilibrium. The mass is gently pulled through a further distance \(A\) and then released so that the mass starts to oscillate along a vertical axis.

   (a) Draw a free-body force diagram FIGURE 1 of the mass when it is passing through the equilibrium position. Given that the acceleration due to gravity \(g\), write down an equation which fully describes this equilibrium of forces. Call this EQUATION 1.

   (b) Draw a diagram FIGURE 2 showing the position of the oscillating mass when it is below the equilibrium position. Label the displacement \(y\) from the equilibrium position. The size of the displacement \(y\) is \(\delta l\).

   (c) State the value of the total extension of the spring when the mass is in the position described in FIGURE 2.

   (d) Referring to FIGURE 2, write down an equation to describe the magnitude \(F\) of the resultant force \(F\) acting on the mass when it is at a given displacement \(y\) from the equilibrium position. Call this EQUATION 2.

   (e) Use EQUATION 1 and EQUATION 2 to obtain a simpler expression for the resultant force \(F\) in terms of \(\delta l\).

   (f) Modify your answer to (v) to obtain a relationship between \(F\) and \(y\). Explain your reasoning.

   (g) The resultant force \(F\) is a restoring force. Explain.

   (h) Derive an equation describing the relationship between the acceleration \(a\) and the displacement \(y\) from the equilibrium position.

   (i) Explain why the equation relating \(a\) to \(y\) suggests that the oscillations of the mass-spring system are simple harmonic.

   (j) The general relationship between the acceleration \(a\) and the displacement \(y\) for a simple harmonic oscillator is \(a = -\omega^2 y\) where \(\omega\) is the angular frequency. Use this equation to find the value of \(\omega\) for this mass-spring system in terms of \(m\) and \(k\).

   (k) Express \(\omega\) in terms of the extension \(e\) of the spring when the system is in equilibrium and the acceleration due to gravity \(g\)

   (l) Derive an expression for the periodic time \(T\) of the mass-spring system in terms of \(m\) and \(k\).

5. A 200-g mass is attached to the lower end of a light helical spring hanging from a fixed support and produces an extension of 4.5 cm. The mass is then gently pulled a further distance of 2.5 cm and released. Calculate
(a) the stiffness $k$ of the spring,
(b) the periodic time of the oscillating mass-spring system,
(c) the maximum value of the acceleration during the motion. (Take the acceleration due to gravity $g$ as 10 m s$^{-2}$.

6. A simple harmonic oscillator is an oscillator that is neither driven nor damped. A particular simple harmonic oscillator is oscillating with an amplitude of 0.05 m and a frequency of 5 Hz. Draw graphs to illustrate the variation of

(a) the displacement of the oscillating mass from its equilibrium position with time
(b) the velocity with time
(c) the acceleration of the mass with time

during a period of 0.3 s if at $t=0$ the oscillator is passing through the equilibrium position. Label your graphs properly including numerical values for the maximum displacement, velocity and acceleration.

7. For a simple harmonic oscillator, the relationship between the velocity $v$ of the oscillating mass and the displacement $y$ from the equilibrium position is given by

$$v = \pm \omega \sqrt{A^2 - y^2}$$

where $A$ is the amplitude and $\omega$ is known as the angular frequency and is equal to $2\pi f$.

A body of mass 50 g is oscillating in SHM with an amplitude of 5 cm and a frequency of 40 Hz.

(a) State or calculate the velocity of the oscillating body
   i. at the centre of the motion,
   ii. at the extremities,
   iii. at a position midway between the centre and the extremities

(b) Sketch a graph showing the variation of the velocity $v$ with the displacement $y$ from the equilibrium position for the oscillating mass

(c) State or calculate the kinetic energy of the oscillating body
   i. at the centre of the motion,
   ii. at the extremities,
   iii. at a position midway between the centre and the extremities