Statements and Logic

1. Show that \((p \land q) \lor (\neg p \land \neg q) \iff (p \lor \neg q) \land (\neg p \lor q)\). (Hint: you may use a truth table or use the Distributive Laws.)

2. Let \(p/q\) be defined as \(\neg p \land \neg q\).
   
   (a) Write down the truth table for \(p/q\).
   (b) Show that \(\neg p \iff p/p\) and that \((p \land q) \iff (p/p)/(q/q)\).
   (c) Find a way of expressing \(p \lor q\) by using only the symbol / (but not any of \(\neg, \lor, \land\) etc.).

3. Which of the following statements are true? Write down the negation of each one.
   
   (a) \(\forall y \in \mathbb{R}, \exists x \in \mathbb{R}\) such that \(x^2 = y\).
   (b) There exists an integer \(y\) such that, for all integers \(x, y\) is a factor of \(x\).
   (c) For all rational numbers \(a\) and \(b\), \(a + b\) is also a rational number.

4. Are the following statements true? Find their negation, converse and contrapositive, and decide which are true:
   
   (a) For all real numbers \(x, y\) and \(z\), if \(x = y\) then \(xz = yz\).
   (b) \(\forall a, x, y \in \mathbb{R}, (x > y) \Rightarrow (ax > ay)\).
   (c) For all \(a, b, c \in \mathbb{Z}\), if \(b\) is a factor of \(a\) and \(c\) is a factor of \(a\), then \(bc\) is a factor of \(a\).

5. Find the negation of the following statement about the function \(f\) and the fixed real number \(a\):
   
   \(\forall \epsilon > 0, \exists \delta > 0\) s.t. \(\forall x \in \mathbb{R}, \) if \(|x - a| < \delta\), then \(|f(x) - f(a)| < \epsilon\).
   
   (This is the precise definition of the statement ‘\(f\) is continuous at \(a\).’)

6. Which (if any) of the following statements about \(x \in \mathbb{Z}\) are true:
   
   (a) \(\forall x, [x\text{ is even }\lor x\text{ is odd}]\) (b) \([\forall x, x\text{ is even}] \lor [\forall x, x\text{ is odd}]\)
   (c) \(\exists x, [x\text{ is even }\land x\text{ is odd}]\) (d) \([\exists x, x\text{ is even}] \land [\exists x, x\text{ is odd}]\)?

7. There is a barber in Seville who shaves the beards of all those men that do not shave themselves. Who shaves the barber?