UNIVERSITY OF MALTA
FACULTY OF SCIENCE
Department of Mathematics
MAT1801 Mathematics for Engineers I
Problem Sheet 6

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(1) Using the theorems on limits of sequences and the definition of a limit of a sequence, find the limit as $n \to \infty$, if any, of the following sequences:

$$\frac{\sin n}{n}, \quad \frac{4-2n}{3n+2}, \quad \frac{n^4+1}{n^6}, \quad 2^{\frac{1}{n}}, \quad \frac{n^4+1}{n^2}.$$

(2) Show that the sequence $a_n = \frac{n+2}{n+1}$ is monotonic decreasing and bounded below, and hence show that it converges. Prove that the limit is 1.

(3) Show that the sequence $a_n = \frac{1-n}{n^2+1}$ is monotonic increasing (for $n = 3, 4, 5, \ldots$) and bounded above, and hence show that it converges. Prove that the limit is 0.

(4) The sequence u_n is defined by the recursion formula $u_{n+1} = \sqrt{3u_n}$, $u_1 = 1$. Prove that the sequence is monotonic increasing and bounded above, and hence show that it converges. Prove that the limit is 3.

(5) Using the Ratio test, determine the convergence or otherwise of the following series:

$$\text{(i)} \sum_{n=1}^{\infty} \frac{(n+1)^3}{2^n}, \qquad \text{(ii)} \sum_{n=1}^{\infty} \frac{n^2}{3^n}, \qquad \text{(iii)} \sum_{n=1}^{\infty} \frac{n^n}{n!}.$$

(You may assume that $\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e$.)

(6) Using the integral test, determine the convergence or otherwise of the following series:

$$\text{(i)} \sum_{n=1}^{\infty} \frac{1}{(n+2)^3}, \qquad \text{(ii)} \sum_{n=1}^{\infty} \frac{1}{n^2}.$$

(7) Consider the infinite series

$$1 + 2r + r^2 + 2r^3 + r^4 + 2r^5 + \ldots,$$

where (a) $r = \frac{2}{3}$, (b) $r = -\frac{2}{3}$, (c) $r = \frac{4}{3}$. Show that the ratio test is inapplicable for the above series and using the nth root test, show that the series converges for cases (a) and (b), and diverges in case (c).