Introduction to Graph Theory — Sheet 2

If time permits, all problems will be worked out in class. Those problems marked with a "-" are the easiest.

1. Prove that if T is a tree with maximum degree Δ then it contains at least Δ vertices of degree 1.

2. - Give an example to show that if P is a $[u, v]$-path in a block (that is, a 2-connected graph), then the block does not necessarily contain another $[u, v]$-path internally disjoint from P.

3. Show that a graph all of whose degrees are even cannot have a cutedge.

4. - Let G be a connected graph with $n \geq 3$ (n is the number of vertices of G). Show that if G has a cutedge then it has a cutvertex. Show by an example that the converse is false.

5. - Recall that Theorem III.1 stated that

   $$\kappa(G) \leq \lambda(G) \leq \delta(G)$$

   where κ, λ, δ are, respectively, the connectivity, edge-connectivity and minimum degree. Show, by an example, that these inequalities can be strict.

6. - Let G be connected. How many components are obtained by removing a λ-set of separating edges? How many components can be obtained by removing a κ-set of separating vertices?

7. Let G be a simple graph with $\delta \geq n - 2$. Prove that $\kappa(G) = \delta$. Give an example of a graph with $\delta = n - 3$ and $\kappa < \delta$.

8. Show that

   (a) If G is a k-regular bipartite graph with $k > 0$ and bipartition (X, Y), then $|X| = |Y|$.

   (b) In a k-regular bipartite graph with $k \geq 2$ there can be no cutedge.

9. Show that in a simple graph if $\delta \geq n - 2$ then $\kappa = \delta$. Find a simple graph with $\delta = n - 3$ and $\kappa < \delta$.

10. Show that in a simple graph if $\delta \geq n/2$ then $\lambda = \delta$. Find a graph with $\delta = n/2 - 1$ and $\lambda < \delta$.