UNIVERSITY OF MALTA
FACULTY OF SCIENCE
Department of Mathematics
B.Sc. 2nd Year
May Session 2001

MA112 Groups (1.5 credits) **May 2001**

0900–1100

Answer THREE questions

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1. Let G be a finite group acting on a finite set X. For $x \in X$ let $G(x)$ and G_x denote, respectively, the orbit and the stabiliser of x. Prove that

$$|G| = |G(x)| \cdot |G_x|.$$

Now suppose that G is a p-group, and let X_1 be the set of fixed points of this action. Show that

$$|X_1| = |X| \bmod p.$$

2. Let G be a finite group acting on a finite set X. Let $F(g)$ be the set of elements of X which are fixed by the permutation $\hat{g}$ of X corresponding to $g \in G$ under this action. Prove that the number of orbits of X under this action equals

$$\frac{1}{|G|} \sum_{g \in G} |F(g)|.$$

Find the number of distinct necklaces which can be made from six beads using any of the three colours red, green or blue, if flipping over of the necklace is allowed.

3. State carefully the three theorems of Sylow.

Show that a group of order 56 cannot be simple.

4. (a) Define the action of *conjugacy* of a group G on itself. Which subgroups of G are mapped onto themselves by this action?

Write down the class equation, explaining clearly all the terms involved.

Show that if G is a p-group then $Z(G)| \geq p$.

(b) Find the conjugacy classes of the two dihedral groups D_5 and D_6. Can you give some geometric significance to your results?