UNIVERSITY OF MALTA
FACULTY OF SCIENCE / I.T. BOARD OF STUDIES
Department of Mathematics
B.Sc.,B.Ed.,/ B.Sc.(I.T.) Year II
June Session 2002

MA141 Discrete Methods **8 June 2002**

MA141 (Faculties of Science, Education) students (1.5 credits):
Answer THREE questions. Time allowed TWO hours.

MA141 (I.T. Board) students (1 credit):
Answer TWO questions. Time allowed ONE AND A HALF hours.

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1. (a) Let $A_1, A_2, \ldots, A_n$ be finite sets, and let α_i denote the sum of the cardinalities of the intersections of the sets taken i at a time. Prove that

$$|A_1 \cup A_2 \cup \ldots \cup A_n| = \sum_{i=1}^{n} \alpha_i (-1)^{i+1}.$$

(b) How many integers are there in the range 1 to 999 (inclusive) which are not divisible by any of 2, 5 or 23?

2. (a) Write down the coefficient of x^k in the expansion of $(1-x)^{-n}$ in ascending powers of x.
(b) Find the coefficient of x^{20} in each of the following products:

(i) $(x^2 + x^3 + x^4 + \ldots)^4$

(ii) $(x^2 + x^3 + \ldots + x^{12})^4$

(iii) $(x^2 + x^3 + x^4 + \ldots)^3(x + x^2 + \ldots + x^6)$

3. (a) A loan of Lm3000 is taken from a bank. After a year, and at the end of every subsequent year, a repayment of LmP is effected. Moreover, at the end of every year the bank charges interest at the rate of 1 per cent of the amount owed during that year.

Let A_n denote the amount owed to the bank at the end of the nth year (therefore $A_0 = 3000$). Obtain and solve a recurrence relation for A_n.
(b) Solve the recurrence relation

$$a_{n+2} - 5a_{n+1} + 6a_n = n.5^n \qquad (n \geq 0)$$

given that $a_0 = a_1 = 0$.

4. (a) Let $p(n)$ denote the number of partitions of the positive integer n and let $p(n|\mathcal{P})$ denote the number of partitions satisfying a given property $\mathcal{P}$.

(i) Write down the generating function for $p(n)$.

(ii) Let $k \geq 1$ be fixed, let $\mathcal{P}_1$ be the property "No part in the partition appears more than k times" and let $\mathcal{P}_2$ be the property "No part in the partition is divisible by $k+1$". Prove that $p(n|\mathcal{P}_1) = p(n|\mathcal{P}_2)$.

(b) Let $S(n,k)$ denote the number of ways of partitioning an n-set into k parts. Write down a recurrence relation for $S(n,k)$ and use this relation to find all values of $S(n,k)$ for $1 \leq k \leq n \leq 5$.

How many ways are there to distribute 5 distinguishable balls amongst 6 distinguishable boxes such that exactly three of the boxes are nonempty? [Hint: For each choice of the three nonempty boxes, each distribution is a surjection from a 5-set to a 3-set.]