UNIVERSITY OF MALTA
FACULTY OF SCIENCE
Department of Mathematics
B.Ed./B.Sc./B.Sc.(IT) Year II
June Session 2003

MAT2402 Networks **2 ECTS Credits**

Answer TWO questions. Time allowed ONE AND A HALF hours.

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1. (a) Define the terms *matroid* and *weighted matroid*.

Let G be a connected graph and let S be the set of edges of G. Also, let the independent subsets of S be all those sets of edges which do not contain a cycle, that is, forests of G. Show that this defines a matroid. [You may assume that the number of components of a forest on n vertices and k edges is $n-k$.]

Describe a greedy algorithm for finding an independent subset with maximum weight in a weighted matroid and prove that this algorithm does actually find an independent subset of maximum weight.

Indicate very briefly how all the above shows that Kruskal's algorithm does find a minimum weight spanning tree in a connected graph.

(b) For some optimisation problems, no algorithm which always gives an optimal solution in reasonable time has yet been found. In these cases, heuristic algorithms are often employed.

Illustrate these statements using examples of optimisation problems which you have studied and indicating how the analysis of heuristic algorithms differs in scope from that of algorithms which give optimal solutions.

2. (a) An individual who lives in town A and works in town F seeks a car route that will minimise his morning driving time. He has recorded driving times (in minutes) along major roads between different intermediate locations. This data is shown in the tables below, where the entry ∞ signifies that there is no direct route between the corresponding locations.

Determine the best commuting route for this person.

	A	B	C	D	E	F
A	$-$	18	∞	32	∞	∞
B		$-$	12	28	∞	∞
C			$-$	17	∞	32
D				$-$	4	17
E					$-$	11

(b) A toy company wants to introduce a new product for Christmas. The tasks required before the product can be made available in the shops, together with estimates for their durations, are:

Activity	Predecessors	Mean	Variance
A=obtain raw materials	-	6	1.78
B=train workers	-	9	1.78
C=publicity campaign	A, B	8	2.78
D=set up prod line	A, B	7	4.00
E=produce product	D	10	0.44
F=obtain orders	C, E	12	1.00

[*Note:* "Mean" signifies mean duration and "Variance" signifies variance of the durations, in days.]

When should work start on the project so that there is a 99% chance that the new product is in the stores by 1 December?

Comment on the assumptions made in obtaining this result.

3. (a) The table below gives the airline distances in hundreds of miles between the six cities: London, Mexico City, New York, Paris, Beijing and Tokyo:

	L	MC	NY	Pa	Pe	T
L	-	56	35	2	51	60
MC		-	21	57	78	70
NY			-	36	68	68
Pa				-	52	61
Be					-	13

Find a shortest spanning tree connecting these six cities.

A traveller wishes to visit each of these cities once, returning to his starting point. Use the spanning tree obtained above to obtain an itinerary visiting each city exactly once and ending at the fist city.

If L_o is the length of a shortest itinerary meeting the traveller's requirements, deduce from the itinerary you have obtained a lower bound for L_o.

[You may assume that the above distances satisfy the triangular inequality.]

(b) It is required to find a minimum weight spanning *path* in a graph G which has wieghts on each of its edges. Show how this can be done by transforming the problem into a travelling salesman problem.