UNIVERSITY OF MALTA
FACULTY OF SCIENCE
Department of Mathematics
B.Ed./B.Sc./B.Sc.(IT) Year II
January Session 2004

MAT2402 Networks—2 ECTS Credits **24 January 2004**

Time: 0915–1045

Answer TWO questions. Time allowed ONE AND A HALF hours.

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1. (a) Define the terms *matroid* and *weighted matroid*.

Describe a greedy algorithm for finding an independent subset with maximum weight in a weighted matroid and prove that this algorithm does actually find an independent subset of maximum weight.

(b) Solve the following two problems:

(i) Find a minimum weight spanning tree in the graph on five vertices $1, 2, 3, 4, 5$ in which the weight of the edge joining vertices i and j is given by the ij-th entry in the following matrix (a dash indicates no edge):

$$\begin{bmatrix} - & 1 & 2 & 4 & 7 \\ 1 & - & 3 & 6 & 7 \\ 2 & 3 & - & 5 & 7 \\ 4 & 6 & 6 & - & 7 \\ 7 & 7 & 7 & 7 & - \end{bmatrix}.$$

(ii) A set of jobs is given. There are no precedence rules governing the jobs but each job has a deadline (in days) and a penalty (in Liri) associated with it and if a job is finished after the deadline then the penalty has to be paid. Every job has a duration of one day. There is only one worker to do all the jobs and no two jobs can be done simultaneously, therefore a schedule is a sequencing of the jobs in the order in which they are to be carried out. If the jobs and the penalties are as shown in the table below, find a schedule which will minimise the total penalty paid.

Jobs	*A*	*B*	*C*	*D*	*E*	*F*
Deadlines	3	2	1	1	1	5
Penalties	100	80	75	60	60	10

(c) Indicate **very briefly** the matroids involved in each of Questions b(i) and b(ii) which make possible the use of a greedy algorithm.

2. (a) A travelling salesman wants to visit cities A, B, C, D, E, F. He would like to devise an itinerary which will enable him to visit each town exactly once and will take him back to the point of departure. He would like to do this in a way which minimises the total distance travelled. The distances between the cities are given in the following table.

	A	B	C	D	E	F
A	-	13	11	6	9	11
B		-	8	14	3	4
C			-	15	6	6
D				-	15	14
E					-	2
F						-

By first finding a minimum spanning tree for the network of cities *with F excluded*, obtain a lower bound for the salesman's problem.

Now, by finding a minimum spanning tree for the whole network, obtain an itinerary (and give its distance) for the salesman which is not worse than twice the optimal solution. [You may assume that the above distances satisfy the triangular inequality.]

(b) Describe a heuristic algorithm for the Travelling Salesman Problem (with distances obeying the triangular inequality) such that if the length of the cycle given by the algorithm is L and the length of an optimal cycle is L_o then

$$L \leq \frac{3}{2} L_o.$$

3. A system of pipelines carries gas from the source S to the sink T, via intermediate points $A, B, \ldots, F$. Each pipeline has a maximum capacity (in thousands of litres per minute) which cannot be exceeded. Moreover, no accumulation of gas at any of the intermediate points is allowed. At present, a total of 5000 litres per minute is carried by the system from S to T. This flow is shown in the table below. In this table, each pair of numbers in row X and column Y denotes the capacity of the pipeline leading directly from point X to point Y and the flow passing through it (in 1000 litres per minute); a blank entry means that there is no pipeline from X to Y.

Show that this flow can be augmented to a maximum of 6000 litres per minute.

	S	A	B	C	D	E	F	T
S	-	$(3,3)$	$(2,2)$	$(2,0)$	-	-	-	-
A	-	-	-	-	$(5,3)$	$(1,0)$	-	-
B	-	-	-	-	$(1,0)$	$(3,2)$	$(1,0)$	-
C	-	-	-	-	-	$(1,0)$	-	-
D	-	-	-	-	-	-	-	$(4,3)$
E	-	-	-	-	-	-	-	$(2,2)$
F	-	-	-	-	-	-	-	$(4,0)$

It is being proposed that, in order to increase the overall capacity of the system, the capacity of *one* of the pipelines CE, BF or ET is to be increased. Which of these three pipelines would you propose should have its capacity increased? Give your reason.