UNIVERSITY OF MALTA
FACULTY OF SCIENCE
Department of Mathematics
B.Sc. Year III
May Session 2001

MA241 Networks **May 2001**

Answer TWO questions. Time allowed ONE AND A HALF hours.

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1. (a) Define the terms *matroid* and *weighted matroid*.

Let G be a conected graph and let S be the set of edges of G. Also, let the independent subsets of S be all those sets of edges which do not contain a cycle, that is, the forests of G. Show that this defines a matroid. [You may assume that the number of components of a forest on n vertices and k edges is $n - k$.]

Describe a greedy algorithm for finding an independent subset with maximum weight in a weighted matroid.

Indicate very briefly how all the above shows that Kruskal's algorithm does find a minimum weight spanning tree in a connected graph.

(b) The following table gives a list of tasks which need to be carried out. The duration of each task is one day, and there is only one processor to carry them out. Also, there are no precedence rules governing the tasks. In the table d_i denotes the day (deadline) by which task i has to be finished, and w_i denotes the penalty which must be paid if the deadline for task i is exceeded.

Task	1	2	3	4	5	6	7
d_i	4	2	4	3	1	4	6
w_i	70	60	50	40	30	30	10

Letting S be the set of tasks, state, without proof, what definition of independent subsets of S in this case gives a matroid.

Hence use a suitable greedy algorithm to find a schedule which minimises the sum of penalties of the late tasks.

2. (a) A travelling salesman wants to visit cities A, B, C, D, E, F. He would like to devise an itinerary which will enable him to visit each town exactly once and will take him back to the point of departure. He would like to do this in a way which minimises the total distance travelled. The distances between the cities are given in the following table.

	A	B	C	D	E	F
A	-	13	11	6	9	11
B		-	8	14	3	4
C			-	15	6	6
D				-	15	14
E					-	2
F						-

By first finding a minimum spanning tree for the network of cities *with F excluded*, obtain a lower bound for the salesman's problem.

Now, by finding a minimum spanning tree for the whole network, obtain an itinerary (and give its distance) for the salesman which is not worse than twice the optimal solution. [You may assume that the above distances satisfy the triangular inequality.]

(b) A system of pipelines carries gas from the source S to the sink T, via intermediate points $A, B, \ldots, F$. Each pipeline has a maximum capacity (in thousands of litres per minute) which cannot be exceeded. Moreover, no accumulation of gas at any of the intermediate points is allowed. At present, a total of 5000 litres per minute is carried by the system from S to T. This flow is shown in the following table. In this table, each pair of numbers in row X and column Y denotes the capacity of the pipeline leading directly from point X to point Y and the flow passing through it; a blank entry means that there is no pipeline from X to Y.

Show that this flow can be augmented to a maximum of 6000 litres per minute.

	S	A	B	C	D	E	F	T
S	-	$(3,3)$	$(2,2)$	$(2,0)$	-	-	-	-
A	-	-	-	-	$(5,3)$	$(1,0)$	-	-
B	-	-	-	-	$(1,0)$	$(3,0)$	$(1,0)$	-
C	-	-	-	-	-	$(1,0)$	-	-
D	-	-	-	-	-	-	-	$(4,3)$
E	-	-	-	-	-	-	-	$(2,2)$
F	-	-	-	-	-	-	-	$(4,0)$

It is being proposed that, in order to increase the overall capacity of the system, the capacity of *one* of the pipelines CE, BF or ET is to be increased. Which of these three pipelines would you propose should have its capacity increased? Give your reason.

3. A project consists of activities A to J. The expected duration $\mathsf{E}(d)$ and the variance of the duration $\mathrm{Var}(d)$ of each activity is given in the table below. This table also gives the pre-requisites of each of the activities.

Activity	Pre-requisites	$\mathsf{E}(d)$	Var(d)
A	-	3	1/9
B	A	2	1/9
C	A	6	16/9
D	C	2	1/9
E	A	3	4/9
F	B	3	1/9
G	C	5	4/9
H	C	1	1/36
I	F, H	1.5	1/36
J	I	2	1/9

Draw the activity network and find the critical path and its expected duration. What is the probability that the project takes longer than 17 days to be completed? [You may use the normal distribution table which is provided.]

Outline the main assumptions used to obtain your result, and comment on their validity.