UNIVERSITY OF MALTA
FACULTY OF SCIENCE
Department of Mathematics
B.Sc. Year III
June Session 2002

MA241 Networks **17 June 2002**

0900–1030

Answer TWO questions. Time allowed ONE AND A HALF hours.

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1. Describe an algorithm for finding a maximum flow in a basic network and indicate how this algorithm can be used to prove the Maximum Flow Minimum Cut Theorem.

2. (a) The table below gives the airline distances in hundreds of miles between the six cities: London, Mexico City, New York, Paris, Beijing and Tokyo:

	L	MC	NY	Pa	Be	T
L	-	56	35	2	51	60
MC		-	21	57	78	70
NY			-	36	68	68
Pa				-	52	61
Be					-	13

Find a shortest spanning tree connecting these six cities.

A traveller wishes to visit each of these cities once, returning to his starting point. Use the spanning tree obtained above to obtain an itinerary visiting each city exactly once and ending at the first city.

If L_o is the length of a shortest itinerary meeting the traveller's requirements, deduce from the itinerary you have obtained a lower bound for L_o.

(b) An individual who lives in town A and works in town F seeks a car route that will minimise his morning driving time. He has recorded driving times (in minutes) along major roads between different intermediate locations. This data is shown in the tables below, where the entry ∞ signifies that there is no direct route between the corresponding locations.

Determine the best commuting route for this person.

	A	B	C	D	E	F
A	$-$	18	∞	32	∞	∞
B		$-$	12	28	∞	∞
C			$-$	17	∞	32
D				$-$	4	17
E					$-$	11

(c) The table below lists six activities which constitute a project, together with the sequencing requirements and the estimated duration times for each activity.

Find the earliest and latest starting times for each activity, and obtain the least number of days within which the whole project can be completed.

Activity	Pre-requisites	Estimated duration (days)
A	-	2
B	A	3
C	A	4
D	B,C	6
E	-	2
F	E	8

Now suppose that two teams of workers are available, and that each team can work on only one activity at a time and also that both teams cannot work on the same activity. Draw up the critical path schedule of the activities in the project for the two teams of workers. Is this schedule optimal? (Give reasoning.)

3. (a) The seven towns of a small island are connected by a system of one-way roads. The direct-link distances along these roads are given in the first table below, where the ij-th entry denotes the distance in miles along the road from town i to town j, with the entry ∞ signifying that there is no direct link from i to j.

	1	2	3	4	5	6	7
1	0	4	7	1	∞	∞	∞
2	∞	0	4	∞	5	∞	∞
3	∞	∞	0	3	3	1	∞
4	∞	∞	∞	0	∞	∞	2
5	∞	∞	∞	∞	0	3	∞
6	2	∞	∞	∞	∞	0	∞
7	∞	∞	3	∞	∞	5	0

The next table gives the shortest distances from town i to town j for all ordered pairs (i, j) (except where the entry is a "?"). Complete this table by finding the shortest distances from town 1 to all the other six towns.

	1	2	3	4	5	6	7
1	0	?	?	?	?	?	?
2	7	0	4	6	5	5	8
3	3	7	0	2	3	1	4
4	8	12	5	0	8	6	2
5	5	9	11	6	0	3	8
6	2	6	8	3	11	0	5
7	6	10	3	5	6	4	0

A Citizens' Advisory Office is to be set up. It has not yet been decided whether to build the office in towns 3 or 4. If the populations of the seven towns (in hundreds) are as given in the next table, and if it is estimated that, on average, each member of the population will pay one visit a year to the office, which of the two options should be chosen so as to minimise the expected total yearly distance the population has to travel in order to reach the office?

Town	1	2	3	4	5	6	7
Population ($\times 100$)	12	7	8	6	15	9	9

(b) For some optimisation problems, no algorithm which always gives an optimal solution in reasonable time has yet been found. In these cases, heuristic algorithms are often employed.

Illustrate these statements using examples of optimisation problems which you have studied and indicating how the analysis of heuristic algorithms differs in scope from that of algorithms which give optimal solutions.