UNIVERSITY OF MALTA
FACULTY OF SCIENCE
Department of Mathematics
B.Sc.(Hons.) Year IV
June 2001 Examination Session

Mathematics Elective Paper II **June 2001**

0915–1215

Answer FIVE questions, attempting ONE one question from each of FIVE sections.

Section A: MA341 Combinatorics

1. Suppose that C is a q-ary code of length n, and that C is e-error-correcting. Write down a lower bound for the minimum distance δ in C.

Prove that
$$|C| \leq \frac{q^n}{\sum_{i=0}^{e} \binom{n}{i} (q-1)^i}.$$

Now let C be a linear code. What is the minimum non-zero weight in C?

Let H be a check matrix for C. Prove that any $2e$ columns of H are linearly independent.

Now let C be the binary Hamming code $H(r,2)$, which is the code with check matrix H whose columns are all the distinct non-zero vectors in $\{0,1\}^r$.

What are the length and the dimension of C? Show that C is a perfect 1-error-correcting code.

Let C_1 be the above Hamming code with $r = 3$. Suppose that the probability of an error in the transmission of any digit is p (independently of the other digits), and suppose that a transmitted word is decoded adopting the nearest neighbour procedure. What is the probability (in terms of p) that a received word is decoded correctly?

Now suppose C_2 is the 3-fold repetition code of length 12 and information rate 1/3. Find, in terms of p, the probability that a received word is decoded correctly adopting the nearest neighbour procedure with code C_2.

2. (a) Show that if $(X, \mathcal{B})$ is a t-design then it is also an s-design for $s = 1, 2, \ldots, t-1$. Show also that, as a $(t-1)$-design,
$$\lambda_{t-1} = \lambda_t \cdot \frac{v - t + 1}{k - t + 1}$$
where v is the number of elements of X, k is the size of each block, and, for each s, any s points of X are contained together in exactly λ_s blocks.

(b) Show that $\{0, 2, 3, 4, 8\}$ is a difference set in $\mathbb{Z}_{11}$, and use it to construct a symmetric design. What are the parameters of this design?