UNIVERSITY OF MALTA
FACULTY OF SCIENCE / I.T. BOARD OF STUDIES
Department of Mathematics
B.Sc.(Hons) Year I / B.Sc.(Hons) I.T. Year II
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MAT1401 Discrete Methods (4 ECTS credits) **May 2005**

Answer FOUR questions. Time allowed: 2 hours 30 minutes.

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1. Solve the following recurrence relation

$$a_{n+2} - 5a_{n+1} + 6a_n = K^n \qquad (n \geq 0)$$

given that $a_0 = a_1 = 0$, in the two cases: (i) $K = 3$, (ii) $K = 4$.

2. (a) Let $A_1, A_2, \ldots, A_n$ denote finite sets and let α_i $(1 \leq i \leq n)$ denote the sum of the cardinalities of the intersections of the sets taken i at a time. Write down, and prove, the Inclusion-Exclusion Formula giving $|A_1 \cup A_2 \cup \ldots \cup A_n|$ in terms of the α_i.

(b) Two integers are said to be *relatively prime* if they have no common factors except for the factor 1. How many positive integers less than or equal to 500 are relatively prime to both 13 and 17?

3. (a) Use generating functions to find the number of ways in which a sum of 20 can be obtained when 8 distinct dice are rolled.

(b) Let $S(n, k)$ denote the number of partitions of an n-set into k parts. Show that

(i) $S(n, 1) = S(n, n) = 1$;

(ii) $S(n, k) = S(n-1, k-1) + kS(n-1, k)$;

(iii) $S(n, 2) = 2^{n-1} - 1 \quad (n \geq 2)$.

[*Hint:* For (iii) use induction on n.]

4. Let $p(n)$ denote the number of partitions of the positive integer n and let $p(n|\mathcal{P})$ denote the number of partitions of n having property $\mathcal{P}$. Write down the generating functions of each of the following

(i) $p(n)$;

(ii) $p(n|$ all parts are distinct);

(iii) $p(n|$ all parts are odd);

(iv) $p(n|$ no part appears more than twice);

(v) $p(n|$ no part is a multiple of 3).

Show that

$$p(n|\text{ all parts are distinct}) = p(n|\text{ all parts are odd})$$

and

$$p(n|\text{ no part appears more than twice}) = p(n|\text{ no part is a multiple of 3}).$$

5. A loan of Lm3000 is taken from a bank. After a year, and at the end of every subsequent year, a repayment of LmP is effected. Moreover, at the end of every year the bank charges interest at the rate of 1 per cent of the amount owed during that year.

Let A_n denote the amount owed to the bank at the end of the nth year (therefore $A_0 = 3000$). Obtain and solve a recurrence relation for A_n.

How much should the repayment amount P be equal to if the loan (including all interests) is to be repaid by the end of the third year?