UNIVERSITY OF MALTA
FACULTY OF SCIENCE
Department of Mathematics
B.Sc./B.Sc.(IT) Year II
January/February Session 2005

MAT2402 Networks **Jan/Feb 2005**

Answer TWO questions. Time allowed ONE AND A HALF hours.

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1. (a) Let A be the matrix shown below.

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Explain what is meant by saying that

$$\lambda = 2.99$$

is the principal eigenvalue of A with corresponding eigenvector

$$x = (1.33, 2.96, 3.25, 2.51, 4.25, 1.98, 1.98, 1.66, 1.00)^T.$$

Note: No calculations are required. The transpose is taken so that x is a column vector.

(b) Draw the graph whose adjacency matrix is A (label the vertices $v_1, v_2, \ldots, v_9$ such that v_i corresponds to the ith row—or ith column—of A).

Explain the significance of the eigenvector x in terms of the relative relative "importance" of the graph's vertices.

(c) Relate the above to how a search-engine for the WWW could present, in order of importance, those pages satisfying a user's query.

Describe briefly some other issues involved in the construction of a search-engine for the WWW.

2. (a) Define the terms *matroid* and *weighted matroid*.

(b) Explain what the graphic matroid is (you do not need to show that it is, in fact, a matroid).

Describe the greedy algorithm for finding a maximum weight independent set in a given matroid. Explain briefly why this algorithm must produce a maximum weight independent set.

(c) The following is a list of jobs, all of duration 1 day. The deadline, in days, of job i is shown as d_i, and w_i denotes the penalty which should be paid if a job's execution goes beyond the deadline.

Task	1	2	3	4	5	6	7
d_i	4	2	4	3	1	4	6
w_i	100	75	60	55	40	30	28

Find a schedule for doing the jobs which minimises the total sum of penalties which have to be paid.

Explain briefly what matroid is present in this problem.

3. "Packing problems, scheduling problems and the travelling salesman problem share several common characteristics."

Discuss this statement giving examples to illustrate your arguments.