UNIVERSITY OF MALTA
FACULTY OF SCIENCE / I.T. BOARD OF STUDIES
Department of Mathematics
B.Sc.,B.Ed.,/ B.Sc.(I.T.) Year II
May Session 2001

MA141 Discrete Methods **May 2001**
MCU202 Discrete Methods

MA141 (Faculties of Science, Education) students:
Answer THREE questions. Time allowed TWO hours.

MCU202 (I.T. Board) students:
Answer TWO questions. Time allowed ONE AND A HALF hours.

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1. (a) Let $A_1, A_2, \ldots, A_n$ be finite sets, and let α_i denote the sum of the cardinalities of the intersections of the sets taken i at a time. Prove that

$$|A_1 \cup A_2 \cup \ldots \cup A_n| = \sum_{i=1}^{n} \alpha_i (-1)^{i+1}.$$

(b) Let $S(n,k)$ denote the number of partitions of an n-set into k parts. Write down, without proof, the number of surjections from an n-set to a k-set in terms of n, k and $S(n,k)$.

(c) Show that

$$S(n,k) = \frac{1}{k!} \sum_{i=0}^{k} (-1)^i \binom{k}{i} (k-i)^n.$$

[*Hint:* Consider all functions from an n-set to a k-set $K = \{y_1, \ldots, y_k\}$, and let A_i be the set of functions which do not have y_i in their range. Then count surjections.]

2. Solve the following recurrence relation

$$a_n - 4a_{n-1} + a_{n-2} = 3^n \quad (n \geq 2)$$

given that $a_0 = a_1 = 0$.

3. (a) Write down (without proof) the coefficient of x^k in the power series expansion of $(1-x)^n$.

(b) Ten letters are to be chosen (order not important) using the letters a, b, c. In how many ways can this be done if:

(i) Each of the three letters can be chosen an unlimited number of times?

(ii) The letter a must be chosen at least once (but otherwise an unlimited number of times), the letter b cannot be chosen more than five times, and the letter c can be chosen an unlimited number of times?

(c) Let $g(x) = \sum_{r=0}^{\infty} a_r x^r$ and $f(x) = \sum_{r=0}^{k} x^r$. Write down the coefficient of x^k in the product $f(x)g(x)$ in terms of the coefficients of $g(x)$.

4. (a) Let $p(n)$ denote the number of partitions of the positive integer n and let $p(n|\mathcal{P})$ denote the number of partitions satisfying a given property $\mathcal{P}$. Write down the generating function for $p(n)$.

Let $k \geq 1$ be fixed, let $\mathcal{P}_1$ be the property "No part in the partition appears more than k times" and let $\mathcal{P}_2$ be the property "No part in the partition is divisible by $k+1$". Prove that $p(n|\mathcal{P}_1) = p(n|\mathcal{P}_2)$.

(b) Solve the first order recurrence relation

$$a_n = na_{n-1} + (-1)^n \quad (n \geq 2)$$

given that $a_1 = 0$. What is the value of

$$\lim_{n\to\infty} \frac{a_n}{n!}?$$