MA131—June 1994

1. An urn contains N fair dice ($N > 0$). The probability that $N = i$ is given by $(\frac{1}{2})^i$. These dice are then thrown and the sum of the scores is recorded as S. Find

(i) $\mathbf{P}(N \text{ is even})$;

(ii) $\mathbf{P}(S = 4 | N = 2)$;

(iii) $\mathbf{P}(S = 4 | N = 4)$;

(iv) $\mathbf{P}(S = 4 | N \text{ is even})$;

(v) $\mathbf{P}(S = 4 | N = 3)$, $\mathbf{P}(S = 4 | N = 1)$ and $\mathbf{P}(S = 4)$;

(vi) $\mathbf{P}(N \text{ is even} | S = 4)$.

2. The quadratic equation $x^2 - 2x + b = 0$ is known to have two real distinct roots. The coefficient b is a positive random variable which is uniformly distributed in the permissible range of values. Find the expectation and variance of the larger root of the quadratic.

3. (a) Suppose X coins are tossed, where X is a Poisson random variable with parameter λ. Each coin gives heads with probability p and tails with probability $q = 1 - p$ independently of the others. Let Y be the number of heads obtained. Show that Y is a Poisson random variable with parameter λp.

(b) A box contains n tickets numbered from 1 to n, each ticket getting a distinct number. A set A of m tickets ($m \leq n$) is randomly drawn from the box. Let the random variable S be defined to be the sum of the tickets in A. For $i = 1, \ldots, n$, let the random variable S_i be defined by putting $S_i(A) = 1$ if ticket i is in A and $S_i(A) = 0$ if ticket i is not in A. Find $\mathsf{E}(S_i)$ and hence, using linearity of expectation, find $\mathsf{E}(S)$.